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# Robust Distributed Control of Uncertain Nonlinear Vehicle Platoons*

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***Abstract*— This paper studies the distributed control problem of uncertain vehicle platoons. The vehicle is modelled by a third-order nonlinear longitudinal dynamics. Under general directed information flow topologies, we propose a robust distributed controller embedding adaptive laws to estimate the control gains. By using the Lyapunov method, we show that the developed control law can ensure that all the uncertain nonlinear vehicles in question track a desired speed and maintain a fixed inter-vehicle spacing.**

## I. INTRODUCTION

In the last few decades, platoon control of multiple vehicles has attracted a large amount of attention in the field of systems and control, due to its effectiveness in improving traffic safety, increasing highway throughput and etc.; see [2], [7], [8], [11]. It aims at finding a feedback controller assuring that all vehicles in platoon move at the same speed while maintain a desired inter-vehicle spacing. Many interesting control schemes have been developed; see [4], [6], [12]. Note that, the aforementioned works focus on the linear vehicle dynamics.

Recently, several attempts have been made for the nonlinear vehicle dynamics. For example, [3], [9], [14], [15] used the feedback linearization technique converting the nonlinear vehicle dynamics into a linear one and then constructed linear controllers for the latter linear dynamics. A drawback of this feedback linearization approach is that the developed controller requires to know all parameters in the vehicle dynamics, i.e., the vehicle dynamics can not contain parameter uncertainties. Note that, various uncertainties unavoidably appear in practical vehicles. Thus, how to design robust distributed controllers for uncertain nonlinear vehicle platoon is an interesting and important issue.

In this paper, we will consider the robust distributed control problem of a platoon of nonlinear vehicles in the presence of parameter uncertainties. By constructing a nonlinear distributed controller, we can successfully solve the platoon control problem, i.e., all the vehicles will keep a desired inter-vehicle spacing. Our main contribution is two-fold. On the one hand, for a practical starting point, all the vehicles in the platoon are heterogeneous and certain dynamic compensators are introduced to handle such heterogeneity. On the other hand, by using adaptive control technique, the system parameters in vehicle longitudinal dynamics do not need to be known exactly, giving rise to a robust distributed controller independent of all the vehicle model parameters.

The rest is organized as follows. Section II introduces the formulation of platoon control problem. Section III presents the main result. Finally, Section IV closes the paper.

## II. PROBLEM FORMULATION

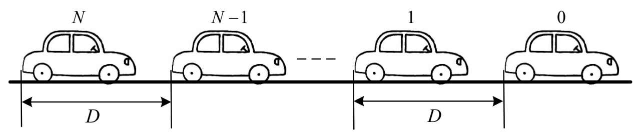


Fig. 1. Platoon of vehicles.

Consider a platoon of $N$ vehicles driving on a straight-line lane; see Fig. 1. It includes a leading vehicle indexed by 0 and $N$ following vehicles indexed from 1 to $N$. Each follower vehicle has the following nonlinear longitudinal dynamics (cf. [14])

$$\begin{aligned}\dot{s}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= \frac{\eta_{Ti}}{m_i l_i}T_i(t) - \frac{c_{Ai}}{m_i}v_i^2(t) - gf_r,\\ \dot{T}_i(t) &= -\frac{1}{\tau_i}T_i(t) + \frac{1}{\tau_i}u_i(t), \quad i = 1, \cdots, N \end{aligned} \tag{1}$$

where $s_i$ is vehicle position, $v_i$ is vehicle velocity and $T_i$ is the actual driving/braking torque, $u_i \in \mathbb{R}$ is the controller input. $g$ is the acceleration due to gravity, $f_r$ is the coefficient of rolling resistance $m_i$ being the vehicle mass, $\eta_{Ti}$ being the mechanical efficiency of driveline, $l_i$ being the tire radius, $c_{Ai}$ being the lumped aerodynamic drag coefficient, and $\tau_i$ being the inertial delay of vehicle powertrain. Throughout this paper, we assume that all the parameters $m_i, f_r, \eta_{Ti}, l_i, c_{Ai}, \tau_i$ are *unknown*.

In the vehicular platoon, the leading vehicle describes the reference trajectory. As in [14], [15], the leading vehicle is assumed to be of constant velocity, i.e., $s_0(t) = v_0 t$, and $v_0$ is an *unknown* constant.

Relating to the platoon, a directed interaction graph $\mathcal{G} := \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ is defined to describe the information flow topology among vehicles, where $\mathcal{V} := \{0, 1, 2, \ldots, N\}$ is the node (vehicle) set, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set with no self-loop, and $\mathcal{A} = [a_{ij}]_{i,j=0}^{N}$ is the weighted adjacency matrix. The set $\mathcal{N}_i := \{j \in \mathcal{V} \mid a_{ij} = 1\}$ is the neighboring set of vehicle $i$. A directed path from vehicle $i_1$ to vehicle $i_k$ is an ordered sequence of distinct vehicles $i_1, \cdots, i_k \in \mathcal{V}$ such that the pairs $(i_1, i_2), \cdots, (i_{k-1}, i_k)$ are edges. A vehicle $i$ is said to be reachable from another vehicle $j$ if there is a

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directed path from vehicle $j$ to vehicle $i$. $\mathcal{G}$ is said to contain a directed spanning tree if there is at least one vehicle, called the root, from which every other vehicle is reachable. The elements of the matrix $\mathcal{A}$ satisfy that $a_{ij} \geq 0$ and $a_{ij} = 1$ if and only if vehicle $i$ can receive the information of vehicle $j$, i.e., $(j, i) \in \mathcal{E}$. The Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]_{i,j=0}^N$ with $l_{ij} = -a_{ij}, i \neq j$ and $l_{ii} = \sum_{j=0}^N a_{ij}$.

Our objective is to design a controller for the platoon to maintain a desired inter-vehicle spacing, i.e.,

$$s_{i-1}(t) - s_i(t) = D, \quad 1 \leq i \leq N \tag{2}$$

where $D$ is the desired constant spacing.

For each $i = 1, \cdots, N$, define

$$e_i := s_i - s_0 + iD \tag{3}$$

which denotes the tracking error between the $i$th vehicle and its desired trajectory. In the platoon, this tracking error is not available for controller design. In practice, determined by the information flow topology, each vehicle $i$ has a neighbor-based measurement as follows

$$e_{mi} := \sum_{j \in \mathcal{N}_i} \big(s_i - s_j + (i-j)D\big). \tag{4}$$

With these preparations, the platoon control problem can be formulated as follows.

**Problem 2.1:** Design a distributed controller of the form

$$\begin{aligned} \dot{\zeta}_i &= h_i(e_{mi}, v_i, T_i, \zeta_i), \\ u_i &= k_i(e_{mi}, v_i, T_i, \zeta_i), \quad i = 1, \cdots, N \end{aligned} \tag{5}$$

such that, for any vehicle initial condition, the closed-loop system has a well-defined solution over $[0, +\infty)$, and moreover the tracking error satisfies

$$\lim_{t \to \infty} e_i(t) = 0, \quad i = 1, \cdots, N. \tag{6}$$

In the following, we list the following assumption as a sufficient condition for solving the platoon control problem.

**Assumption 1:** The information flow topology $\mathcal{G}$ contains a directed spanning tree with the leading vehicle as the root.

Assumption 1 indicates that, there exists a directed path from the heading vehicle to any other following vehicle. This assumption has been widely used as a basic and typical condition in multi-agent systems; see [5], [10], [13]. Let $H$ be the $N \times N$ submatrix obtained by removing the first row and column of the Laplacian matrix $\mathcal{L}$ of $\mathcal{G}$. Under Assumption 1, known from [1], all the eigenvalues of $H$ have positive real parts.

## III. Main Result

In this section, we present a distributed controller for solving Problem 2.1 of the platoon (1). More specific, we establish a two-step procedure. First, we will construct dynamic compensators converting Problem 2.1 into a distributed stabilization problem. Second, the consequent distributed stabilization problem will be solved by a distributed adaptive control law.

### A. Dynamic Compensator Design

Define

$$\begin{aligned} v_i^\star &= v_0, \\ T_i^\star &= \frac{1}{\eta_{Ti}}(c_{Ai} v_i^{\star 2} + m_i g f_r), \\ u_i^\star &= \frac{1}{\eta_{Ti}}(c_{Ai} v_i^{\star 2} + m_i g f_r) \end{aligned} \tag{7}$$

It can be seen that, the above defined $(v_i^\star, T_i^\star, u_i^\star)$ characterizes the necessary steady-state velocity, driving/braking torque, and control input, assuring zero tracking error. Due to the presence of vehicle uncertainty and unknown velocity $v_0$, the information $(v_i^\star, T_i^\star, u_i^\star)$ can not be used in designing the distributed controller. To realize the distributed control design, we first introduce the following dynamic compensator

$$\dot{\hat{v}}_i = -\hat{v}_i + v_i, \quad \dot{\hat{T}}_i = -\hat{T}_i + T_i, \quad \dot{\hat{u}}_i = -\hat{u}_i + u_i \tag{8}$$

to asymptotically reproduce $(v_i^\star, T_i^\star, u_i^\star)$.

Let

$$\begin{aligned} & e_i = s_i - s_0 + iD, \quad \bar{v}_i = v_i - \hat{v}_i, \\ & \bar{T}_i = T_i - \hat{T}_i, \quad \bar{u}_i = u_i - \hat{u}_i \\ & \tilde{v}_i = \hat{v}_i - v_i^\star - e_i, \quad \tilde{T}_i = \hat{T}_i - T_i^\star - \frac{m_i l_i}{\eta_{Ti}} \bar{v}_i, \\ & \tilde{u}_i = \hat{u}_i - u_i^\star - \tau_i \bar{T}_i. \end{aligned}$$

Then, we obtain the following system

$$\begin{aligned} \dot{\tilde{v}}_i &= -\tilde{v}_i + \varphi_i^a(e_i), \\ \dot{e}_i &= \bar{v}_i + \phi_i^a(\tilde{v}_i, e_i), \\ \dot{\tilde{T}}_i &= -\tilde{T}_i + \varphi_i^b(\tilde{v}_i, e_i, \bar{v}_i), \\ \dot{\bar{v}}_i &= \frac{\eta_{Ti}}{m_i l_i} \bar{T}_i + \phi_i^b(\tilde{v}_i, e_i, \tilde{T}_i, \bar{v}_i), \\ \dot{\tilde{u}}_i &= -\tilde{u}_i + \varphi_i^c(\tilde{T}_i, \bar{v}_i, \bar{T}_i), \\ \dot{\bar{T}}_i &= \frac{1}{\tau_i} \bar{u}_i + \phi_i^c(\tilde{T}_i, \bar{v}_i, \bar{T}_i, \tilde{u}_i) \end{aligned} \tag{9}$$

where

$$\begin{aligned} \varphi_i^a &= -e_i, \quad \phi_i^a = e_i + \tilde{v}_i, \\ \varphi_i^b &= -\frac{m_i l_i}{\eta_{Ti}} \bar{v}_i + \frac{\eta_{Ti}}{m_i l_i} T_i^\star \\ &\quad - \frac{c_{Ai}}{m_i} (\bar{v}_i + \tilde{v}_i + v_i^\star + e_i)^2 - g f_r - \bar{v}_i, \\ \phi_i^b &= \frac{\eta_{Ti}}{m_i l_i} (\tilde{T}_i + T_i^\star + \frac{m_i l_i}{\eta_{Ti}} \bar{v}_i) \\ &\quad - \frac{c_{Ai}}{m_i} (\bar{v}_i + \tilde{v}_i + v_i^\star + e_i)^2 - g f_r - \bar{v}_i, \\ \varphi_i^c &= (1 - \tau_i) \bar{T}_i + \tilde{T}_i + \frac{m_i l_i}{\eta_{Ti}} \bar{v}_i, \\ \phi_i^c &= -\frac{1}{\tau_i} (\bar{T}_i + \tilde{T}_i + \frac{m_i l_i}{\eta_{T_i}} \bar{v}_i - \tilde{u}_i). \end{aligned}$$

For the sake of convenience, we further let

$$\begin{aligned} & \tilde{v} = [\tilde{v}_1, \cdots, \tilde{v}_N]^\intercal, \; e = [e_1, \cdots, e_N]^\intercal, \; \tilde{T} = [\tilde{T}_1, \cdots, \tilde{T}_N]^\intercal, \\ & \bar{v} = [\bar{v}_1, \cdots, \bar{v}_N]^\intercal, \; \tilde{u} = [\tilde{u}_1, \cdots, \tilde{u}_N]^\intercal, \; \bar{T} = [\bar{T}_1, \cdots, \bar{T}_N]^\intercal. \end{aligned}$$

Then the system (9) can be rewritten equivalently in the following compact form

$$\begin{aligned}\dot{\tilde{v}} &= -\tilde{v} + \varphi^a(e),\\ \dot{e} &= \bar{v}_i + \phi^a(\tilde{v}, e),\\ \dot{\tilde{T}} &= -\tilde{T} + \varphi^b(\tilde{v}, e, \bar{v}),\\ \dot{\bar{v}} &= M^b \bar{T} + \phi^b(\tilde{v}, e, \tilde{T}, \bar{v}),\\ \dot{\tilde{u}} &= -\tilde{u} + \varphi^c(\tilde{T}, \bar{v}, \bar{T}),\\ \dot{\bar{T}} &= M^c \bar{u} + \phi^c(\tilde{T}, \bar{v}, \bar{T}, \tilde{u}) \qquad (10)\end{aligned}$$

where

$$\begin{aligned}\varphi^a &= [\varphi_1^a, \cdots, \varphi_N^a]^\intercal, \quad \phi^a = [\phi_1^a, \cdots, \phi_N^a]^\intercal,\\ \varphi^b &= [\varphi_1^b, \cdots, \varphi_N^b]^\intercal, \quad \phi^b = [\phi_1^b, \cdots, \phi_N^b]^\intercal,\\ \varphi^c &= [\varphi_1^c, \cdots, \varphi_N^c]^\intercal, \quad \phi^c = [\phi_1^c, \cdots, \phi_N^c]^\intercal,\\ M^b &= \text{diag}\Big(\frac{\eta_{T1}}{m_1 l_1}, \cdots, \frac{\eta_{TN}}{m_N l_N}\Big),\\ M^c &= \text{diag}\Big(\frac{1}{\tau_1}, \cdots, \frac{1}{\tau_N}\Big).\end{aligned}$$

It can be verified that

$$\varphi^a(0) = 0, \quad \phi^a(0,0) = 0, \quad \varphi^b(0,0,0) = 0,$$
$$\phi^b(0,0,0,0) = 0, \quad \varphi^c(0,0,0) = 0, \quad \phi^c(0,0,0,0) = 0.$$

Thus, if we can find a distributed controller solving the distributed stabilization problem of the system (10) in the sense that, the solution of the closed-loop system exists and is bounded over $[0, \infty)$, and moreover, the tracking error satisfies $\lim_{t\to\infty} e(t) = 0$, then, Problem 2.1 can be solved.

At the end of this subsection, we present the following useful lemma.

**Lemma 3.1:** Consider the system (9). All the functions satisfy the following growth conditions.

$$\begin{aligned}&\|\varphi^a(e)\| \le \|e\|, \quad \|\phi^a(e)\| \le \|\tilde{v}\| + \|e\|,\\ &\|\varphi^b(\tilde{v}, e, \bar{v})\| \le \gamma_1 \cdot (\rho(\|\tilde{v}\|) + \rho(\|e\|) + \rho(\|\bar{v}\|)),\\ &\|\phi^b(\tilde{v}, e, \tilde{T}, \bar{v})\| \le \gamma_2 \cdot (\|\tilde{T}\| + \rho(\|\tilde{v}\|) + \rho(\|e\|) + \rho(\|\bar{v}\|)),\\ &\|\varphi^c(\tilde{T}, \bar{v}, \bar{T})\| \le \gamma_3 \cdot (\|\tilde{T}\| + \|\bar{v}\| + \|\bar{T}\|),\\ &\|\phi^c(\tilde{T}, \bar{v}, \bar{T}, \tilde{u})\| \le \gamma_4 \cdot (\|\tilde{T}\| + \|\bar{v}\| + \|\bar{T}\| + \|\tilde{u}\|) \qquad (11)\end{aligned}$$

where the constants $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \ge 1$ rely on the system parameters and thus are unknown, and $\rho(s) := (1 + s)s$.

### *B. Distributed Stabilizer Design*

Before further proceeding, for each vehicle $1 \le i \le N$, let

$$\begin{aligned}&\epsilon_i = \bar{v}_i + k_0 e_{mi}, \qquad (12)\\ &\varepsilon_i = \bar{T}_i + k_{1i}(1 + \epsilon_i^2)^2 \epsilon_i, \quad \dot{k}_{1i} = (1 + \epsilon_i^2)\epsilon_i^2, \quad k_{1i}(0) \ge 1\end{aligned}$$

where $k_0 \ge 1$ is a fixed constant.

We are ready to present the following main result.

**Theorem 3.1:** Under Assumption 1, we can construct a distributed controller of the form

$$\bar{u}_i = -k_{2i}\varepsilon_i, \quad \dot{k}_{2i} = \varepsilon_i^2, \quad k_{2i}(0) \ge 1 \qquad (13)$$

with (12) that solves the distributed stabilization problem of the system (9). Moreover, a distributed controller consisting of (8) and (13) with (12) solves Problem 2.1.

**Remark 3.1:** Note that, in the proposed distributed controller, two adaptive gains $k_{1i}$ and $k_{2i}$ are introduced and can dynamically estimate the necessary control gains relying on the vehicle model parameters. Thus, our method can effectively avoid computation burden of control gain, and allow the vehicle platoons to contain arbitrary large uncertainties.

## IV. CONCLUSION

We have addressed the distributed control problem for nonlinear vehicle platoons. All the vehicles in the platoon have non-identical and uncertain longitudinal dynamics. For each information flow topology containing a directed spanning tree with the heading vehicle as the root, we developed a distributed controller that is robust and independent of all the vehicle model parameters.

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