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Communication-Saving Design by Stochastic Event Triggers

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EXTENDED ABSTRACT

In recent two decades, the technology of wireless communication network grew rapidly and has been applied to numerous fields. This technology also benefits the field of control: by introducing the wireless communication, more complex and flexible control systems are realized. In a wireless control system, accompanied with the convenience brought by the wireless communication, the communication between system components often consumes a majority of power. Meanwhile, the system may spread widely in a terrain and the components may be supplied by local unrechargeable battery. Hence, it is important to save the communication resources. One direction is to schedule the communication of the system components. Usually, the schedule is well designed before the system process. To sufficiently use the real-time information, the event-triggered scheduling is proposed, which further saves the sensor communication [1]. The mechanism is briefly explained as follows. The triggering condition usually has the form of

$$||y_k - b|| \leq \epsilon,$$

where y_k is the measurement at the sampled time k, b is a pre-specified real vector, and ϵ is a small real number. Once this triggering condition is satisfied, the sensor could save the transmission of y_k while the component which the sensor communicates with is able to know that y_k is close to the value b. This type of trigger is called *deterministic trigger*, since the triggering condition has no uncertainty [1], [2]. However, at the receiver side, the estimation of the measured states based on the measurements cannot preserve the Gaussian property anymore, as the triggering condition causes a truncated probability density function. This problem brings additional complexity in the calculation of the estimates. A *stochastic event trigger* is proposed by Han et al. [3], [4], which smartly addresses the problem mentioned above. The stochastic event trigger not only has the similar function as a deterministic trigger, but also preserves the Gaussian property of the estimation.

In this paper, we consider to design the sensor communication which best saves the communication resources using the stochastic event triggers. In many applications, the accurate estimation performance is not necessary, but long lifetime of the system and low equipment cost are urgently demanded. For example, in the field of environment monitoring, a large amount of sensors are used to monitor the environment parameters, which may be distributed in the wild and be battery charged. The estimation of the monitored environment parameters need not to be exactly accurate, but the sensor power should be best saved to maintain a long lifetime. Meanwhile, since a large number of sensors are used, the cost of producing a sensor is demanded to be cheap, which requires simple equipment embedded in the sensor. The study in this paper is towards those applications.

We summarize the contribution of this paper as follows:

1) We propose a framework of the communication-saving design. We pre-specify a set of the stochastic event triggers for the sensor. When one of the triggers is triggered, the identity of the corresponding trigger is sent to the estimator; otherwise nothing is sent. In this way, continuous communication is avoided; meanwhile, the identity of the trigger only needs a few bits, which allows the communication can be realized by very simple signal transmission equipment.
2) We figure out that one stochastic event trigger is triggered and the sensor sends the corresponding identity of the trigger to the estimator is equivalent to that a virtual sensor sends a certain measurement to the estimator. This equivalence facilitates the analysis of the estimation performance and the succeeding design task.
3) A design problem subject to specific requirements is considered, which demonstrates the proposed design framework.

I. FRAMEWORK OF COMMUNICATION-SAVING DESIGN

In this section, we present the general model of the system. We first give the formulations of the process and sensors; then we propose a novel design of the communication from the sensor to the estimator using the event-trigger mechanism, which consumes the most saved communication resource; last we describe the task of the estimator.

A. State Process and Sensor

The system to be studied is shown in Fig. 1. The state of a single dynamic process is measured by a sensor:

$$x_{k+1} = Ax_k + w_k, \quad (1)$$
$$y_k = Cx_k + v_k. \quad (2)$$

In the above equations, $x_k \in \mathbb{R}^n$ is the process state of time k and $y_k \in \mathbb{R}^m$ is the measurement of the sensor. $\{w_k\}$ and $\{v_k\}$ are zero-mean white Gaussian noise processes, satisfying $\boldsymbol{E}[w_k w_j'] = \delta_{kj}Q$ $(Q \geq 0)$, $\boldsymbol{E}[v_k v_j'] = \delta_{kj}R_i$ $(R_i > 0)$, and $\boldsymbol{E}[w_k v_j'] = 0$, $\forall j, k$. The initial state x_0 is assumed Gaussian with distribution $\mathcal{N}(0, \Sigma_0)$. It is also assumed uncorrelated with w_k and v_k, $\forall k$. Let the pair $(A, Q^{\frac{1}{2}})$ be controllable and (C, A) be detectable.

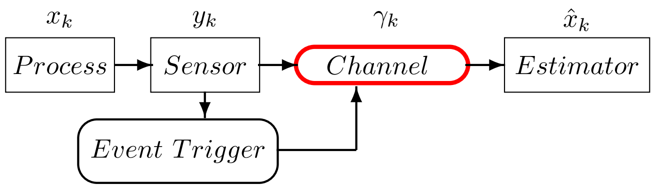

Fig. 1. The system diagram.

B. Saved Communication by Stochastic Event Triggers

After taking measurements of the process states, the sensor needs to transmit its data to a remote estimator. In an ordinary framework, the sensor usually sends the quantized value of the measurements via a communication channel at each sampled time instances. This manner guarantees accurate estimation while consumes great communication resource. However, in many applications, communication resource is scarce, while the performance of the estimator is not required to be that accurate. The scenarios have been discussed in the introduction. The goal of this paper is subject to those scenarios. Instead of sufficient communication, we make use of the mechanism of event triggers and propose a novel design for the sensor communication, which consumes the most saved communication resource.

Define the trigger function as follows

$$\phi(y,b) = \exp\left(-\frac{1}{2}(y-b)'Y(y-b)\right). \quad (3)$$

The sensor is assigned n vectors: $b_1, b_2, ..., b_n \in \mathbb{R}^m$. The way how to choose b_i is to be proposed in the next section. Define γ_k as the *communication decision variable*. At each time k, after obtaining y_k, the sensor generates a random variable ζ_k, which is uniformly distributed within the interval $[0,1]$, and then obtain γ_k as follows:

$$\gamma_k = \begin{cases} i, & \text{if } \zeta_k \leq \phi(y_k, b_i), \\ 0, & \text{otherwise.} \end{cases}$$

If $\zeta_k \leq \phi(y_k, b_i)$ are satisfied for more than one i, we only choose one of them.

The sensor communicates with the estimator according to the following rules: if $\gamma_k = 0$, the sensor does not send anything to the estimator; if $\gamma_k = i$, the sensor sends the binary code of i using corresponding bits.

Remark 1 (The saved communication): The advantage of this sensor communication design is that it greatly saves the communication resource and is easy to install. Firstly, the sensor does not have to do the data transmission at each sampled time instance. Secondly, the data transmitted by the sensor only requires several bits. Compared with the general scenario that the sensor sends the quantized value of the measurement, which usually requires a larger bits packet, the way we propose saves more communication resource. Thirdly, the communication of the sensor can be realized by simple equipment, since only packets of short bits are to be sent.

On the other hand, the trade-off of the saved communication is the sacrificed estimation performance. Hence, this paper faces to those applications where the estimation performance is less important but the communication resource is relatively scarce. For these applications, the result in this paper may be found applicable.

C. Estimator

Define the information set of the estimator at time k as

$$\mathcal{I}_k \triangleq \{\gamma_1, \gamma_2, ..., \gamma_k\}. \quad (4)$$

Further define the estimates as

$$\hat{x}_k^- \triangleq \boldsymbol{E}[x_k|\mathcal{I}_{k-1}], \quad (5)$$
$$\hat{x}_k \triangleq \boldsymbol{E}[x_k|\mathcal{I}_k], \quad (6)$$

and the associated error covariances as

$$P_k^- \triangleq \boldsymbol{E}[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)'|\mathcal{I}_{k-1}],$$
$$P_k \triangleq \boldsymbol{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)'|\mathcal{I}_k].$$

The task of the estimator is to calculate $\hat{x}_k^-$, $\hat{x}_k$, P_k^-, and P_k at each time k.

Remark 2: The typical Kalman filter is used to calculate

$$\hat{x}_k \triangleq \boldsymbol{E}[x_k|\boldsymbol{Y}_k],$$
$$P_k \triangleq \boldsymbol{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)'|\boldsymbol{Y}_k],$$

where $\boldsymbol{Y}_k = \{y_1, y_2, ..., y_k\}$. In this case, the Kalman filter is not applicable, since the estimator receives no sensor measurements.

II. Communication-Saving Design by Stochastic Event Triggers

In this section, we investigate the detailed design of the communication-saving model.

A. Estimation Process

First we find how the estimator works. At each time k, the estimator first calculate the prediction $\hat{x}_k^-$ and the associated error covariance P_k^-:

$$\hat{x}_k^- = A\hat{x}_{k-1}, \quad (7)$$
$$P_k^- = AP_{k-1}A' + Q. \quad (8)$$

When $\gamma_k = 0$, i.e., the estimator received nothing from the sensor, it simply use the prediction values as its final estimate:

$$\hat{x}_k = \hat{x}_k^-, \quad (9)$$
$$P_k = P_k^-. \quad (10)$$

When $\gamma_k = i$, we present the following result.

Theorem 1 (Virtual Sensor): When $\gamma_k = i$, it is equivalent for the estimator to receiving from a virtual sensor

$$\tilde{y}_k = Cx_k + \tilde{v}_k, \quad \tilde{v}_k \sim \mathcal{N}\left(0, R + Y^{-1}\right) \quad (11)$$

a measurement $\tilde{y}_k = b_i$. Correspondingly, the estimator computes $\hat{x}_k$ and P_k as follows:

$$K_k = P_k^- C'(CP_k^- C' + R + Y^{-1})^{-1}, \quad (12)$$

$$\hat{x}_k = \hat{x}_k^- + K_k(b_i - C\hat{x}_k^-), \quad (13)$$

$$P_k = P_k^- - K_k C P_k^-. \quad (14)$$

Proof: The proof is in the appendix. ■

Theorem 1 reveals the advantage of the event-triggered communication. The sensor only sends the simple code γ_k rather than the measurement y_k, but for the estimator it is equivalent to receiving the measurement from a certain sensor.

Remark 3: Through Theorem 1, we can see that the proposed design of event-triggered communication works similarly as a quantizer, while it allows the estimator to preserve the Gaussian property like running a Kalman filter, which maintains the simplicity of the estimation process.

B. Performance Analysis

After we propose the design of event-triggered communication, we need to analyze the performance of the system under the proposed design. In this section, we assume that A is stable.

First, we study the communication rate of the channel. Define λ_k as the communication rate at time k. We have

$$\lambda_k = \Pr(\gamma_k > 0). \quad (15)$$

Lemma 1: A Gaussian random variable y has the probability distribution $\mathcal{N}(\hat{y}, \Pi)$. Then the expectation of the function

$$V(y) = \exp\left(-\frac{1}{2}(y-b)'Y(y-b)\right)$$

is

$$\boldsymbol{E}V = \frac{(2\pi)^{\frac{m}{2}}}{|Y|^{\frac{1}{2}}}\phi(b; \hat{y}, \Pi + Y^{-1}),$$

where the function $\phi(x; \mu, \Sigma)$ is the probability distribution of Gaussian random variable with mean μ and covariance Σ.

Proof: Direct but tedious calculation can obtain the result. ■

Theorem 2: The communication rate λ

$$\begin{aligned}\lambda &\triangleq \lim_{k\to\infty} \lambda_k \\ &= 1 - \prod_{i=1}^{n}\left(1 - \frac{(2\pi)^{\frac{m}{2}}}{|Y|^{\frac{1}{2}}}\phi(b_i; 0, \Pi + Y^{-1})\right),\end{aligned}$$

where $\Pi = C\Sigma C' + R$ and Σ is the solution to $\Sigma = A\Sigma A' + Q$.

Proof: Since A is stable, the solution to $\Sigma = A\Sigma A' + Q$ exists. Hence, x_k has a limit distribution of $\mathcal{N}(0, \Sigma)$. Then the limit of y_k's distribution is $\mathcal{N}(0, \Pi)$.

We have

$$\begin{aligned}\Pr(\gamma_k = i) &= \Pr\left(\zeta_k \le \exp\big(-\frac{1}{2}(y-b)'Y(y-b)\big)\right) \\ &= \boldsymbol{E}\big[\exp\big(-\frac{1}{2}(y-b)'Y(y-b)\big)\big].\end{aligned}$$

Applying Lemma 1 we can obtain the result. ■

Second, we study the estimation performance of the estimator. Since the estimator receives "measurements" from the virtual sensors intermittently, the estimation error covariance P_k depends on γ_k and hence is stochastic. We consider the performance of $\boldsymbol{E}P_k$ instead to eliminate the uncertainty of communication. Define the Lyapunov operator as follows:

$$h(X) \triangleq AXA' + Q, \quad (16)$$

and let

$$\overline{R} = R + Y^{-1}. \quad (17)$$

Consider the modified algebraic Riccati equation (MARE)

$$g(X) = h(X) - \lambda h(X)C'\big(Ch(X)C' + \overline{R}\big)^{-1}Ch(X). \quad (18)$$

We use this MARE to construct a series $\{V_k\}$: $V_0 = P_0$, $V_k = g(V_{k-1})$.

Lemma 2: $\boldsymbol{E}P_k$ is bounded by V_k:

$$\boldsymbol{E}P_k \le V_k. \quad (19)$$

Proof: The proof is similar to the one of Theorem 4 in [5]. Hence, we omit the proof. ■

Since A is stable, V_k will always converge to a steady state. Let

$$\lim_{k\to\infty} V_k = \overline{V}. \quad (20)$$

C. The Design of Event Triggers

In the previous section, we reveal how the event-triggered communication affects the performance of the system, which allows us to investigate the design task in this subsection. We have to tackle the following three facts:

1) Bits to present γ_k,
2) Communication rate λ,
3) Estimation performance $\overline{V}$.

We notice the coupling of the three quantities as follows. The bits of γ_k is determined by the number of the event triggers (the number of b_i). The communication rate λ is determined by b_i and Y, where a larger Y (in the sense of positive definite) causes a smaller probability to trigger the triggering conditions, and hence a smaller communication rate. The upper bound of estimation error covariance $\overline{V}$ is determined by λ and Y: larger λ and larger Y leads to smaller $\overline{V}$. Based on the framework, many problems can be investigated. Hence, we need to tune these quantities to achieve the design demands in specific problem scenarios.

In this subsection, we propose one problem to demonstrate this communication-saving design. Consider the estimation performance needs to satisfy

$$\mathrm{Tr}(\overline{V}) \le \mathcal{P}, \quad (21)$$

where $\mathcal{P}$ is a specified performance level. Moreover, the communication rate should be no larger than a given α:

$$\lambda \le \alpha. \quad (22)$$

We intend to design b_i and Y to realize the goal that the bits of γ_k are minimized.

To solve the problem, we first fix the value of λ: $\lambda = \alpha$. If we want fewer bits of γ_k, we would like to obtain a relatively small Y in the sense of positive definite. We propose the following optimization problem:

Problem 1:

$$\begin{aligned} \min \quad & \mathrm{Tr}(Y) \\ \text{s.t.} \quad & \mathrm{Tr}(\overline{V}) \le \mathcal{P}. \end{aligned}$$

Since $\overline{V}$ is implicit, we need to do transformation of the problem. It is simple to see that this problem is equivalent to

Problem 2:

$$\begin{aligned} \min \quad & \mathrm{Tr}(Y) \\ \text{s.t.} \quad & \mathrm{Tr}(X) \le \mathcal{P}, \\ & X > g(X). \end{aligned}$$

Define

$$\begin{aligned} \psi(L, X) &= h(X) + \lambda^2 L\big((1+q)(Ch(X)C' + \overline{R})\big)L' \\ &\quad -\lambda h(X)H'L' - \lambda LHh(X). \end{aligned}$$

where $q = \frac{1-\lambda}{\lambda}$. Furthermore, we have the following result.

Lemma 3: The following statements are equivalent:

(1) $\exists X \succ 0$, such that $X \ge g(X)$,
(2) $\exists X \ge 0$ and L, such that $X \ge \psi(L, X)$ holds.

Proof: The proof is similar to the one of Theorem 5 of [5]. ■

Lemma 4: The following statements are equivalent:

(1) $\exists X \ge 0$, such that $X \ge \psi(L, X)$,
(2) $\exists W \ge 0$ and Z, such that

$$\begin{bmatrix} W & M_1A & M_1 & M_2A & M_2 & M_3 & M_3 \\ A'M_1' & W & & & & & \\ M_1' & & Q^{-1} & & & & \\ A'M_2' & & & W & & & \\ M_2' & & & & Q^{-1} & & \\ M_3' & & & & & R^{-1} & \\ M_3' & & & & & & Y \end{bmatrix} \ge 0.$$

where $M_1 = YA - \lambda ZCA$, $M_2 = \sqrt{\lambda(1-\lambda)}(Y - ZC)$, and $M_3 = \sqrt{\lambda}Z$.

Moreover, for W satisfying the inequality in (2), $X = W^{-1}$ is a solution to the inequality in (1). It is also true conversely.

Proof: By using Schur decomposition, which is similar to the one of Theorem 5 of [5], the argument can be proved. ■

Hence, Problem 23 is equivalent to the following problem.

Problem 3:

$$\begin{aligned} \min_{X,W,Z} \quad & \mathrm{Tr}(Y) \\ \text{s.t.} \quad & X \le \mathcal{P}, \\ & \begin{bmatrix} X & I \\ I & W \end{bmatrix} \succeq 0, \end{aligned}$$

$$\begin{bmatrix} W & M_1A & M_1 & M_2A & M_2 & M_3 & M_3 \\ A'M_1' & W & & & & & \\ M_1' & & Q^{-1} & & & & \\ A'M_2' & & & W & & & \\ M_2' & & & & Q^{-1} & & \\ M_3' & & & & & R^{-1} & \\ M_3' & & & & & & Y \end{bmatrix} \ge 0.$$

This problem is convex and can be solved efficiently by numerical algorithm.

After solving Y, we need to obtain the proper number and values of b_i by heuristic tuning, such that the communication rate is achieved.

Remark 4: In this section, we have to assume that A is stable. If A is unstable, the measurements y_k may be unbounded, and the event triggers with pre-specified b_i's cannot be triggered, which destroys the system performance. This is due to the simplicity of sensor equipment, which is unable to carry too complicated tasks. When the sensor is more powerful, for example it has local computation capacity, it can manage more complicated tasks.

In practical applications, since the dynamic of the process is usually controlled to be stable, our proposed design is able to serve a large class of systems.

REFERENCES

[1] L. Shi, K. H. Johansson, and L. Qiu, "Time and event-based sensor scheduling for networks with limited communication resources," in *Proceedings of the 18th International Federation of Automatic Control*, Roman, Italy, 2011, pp. 13 263–13 268.
[2] J. Wu, Y. Yuan, H. Zhang, and L. Shi, "How can online schedules improve communication and estimation tradeoff?" *IEEE Transactions on Signal Processing*, vol. 61, no. 7, pp. 1625–1631, 2013.
[3] D. Han, Y. Mo, J. Wu, B. Sinopoli, and L. Shi, "Stochastic event-triggered sensor scheduling for remote state estimation," in *Proceedings of the 52th IEEE Conference on Decision and Control*, Florence, Italy, 2013, pp. 6079–6084.
[4] D. Han, Y. Mo, J. Wu, S. Weerakkody, B. Sinopoli, and L. Shi, "Stochastic event-triggered sensor schedule for remote state estimation," *IEEE Transactions on Automatic Control*, vol. 60, no. 10, pp. 2661–2675, 2015.
[5] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. Jordan, and S. Sastry, "Kalman filtering with intermittent observations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1453–1464, 2004.