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String stability for predecessor following platooning over lossy communication channels

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***Abstract*— We study a platooning scheme where the communication between agents is made through lossy channels. Each agent is modelled as a discrete-time LTI system controlled by a discrete-time LTI controller. The lossy channels are modelled using Bernoulli processes that represent random data dropouts. We consider a scheme that forces inter-vehicle spacings that increase with agents velocities. This is known as a constant time headway spacing policy, which has been shown to provide string stability for predecessor-following architectures. We analyse how the lossy channels impact the platoon string stability.**

I. Introduction

The study of interconnected agents has been an important topic of research in the last couple of decades (see [1], [2], [3], and the references therein). The main reasons behind it are the benefits that automating can have in the performance and safety of transportation systems and other applications [4], [5], [6]. However, several theoretical and practical issues arise if an increasing number of dynamical systems are connected. For example, a one dimensional platooning problem [7], where every agent of a string of vehicles is locally controlled, may exhibit disturbance amplification along the string when a vehicle only uses the relative distance to its predecessor as the controller input. This is associated to the *string stability* concept [5]. String instability is unavoidable in some settings and it has been shown to be caused by a Bode-type fundamental limitation [7].

Two common LTI solutions to achieve string stability involve the use of either, the broadcast of the leader state to every follower [7], [8], or forcing an inter-vehicle spacing that increases with the speed of each agent. The latter is commonly referred to as adding a *constant time headway* to the desired inter-vehicle spacings [9], [10]. Moreover, the interconnection of dynamical systems is subject to the limitations imposed by communication channels. Even more so when the systems correspond to moving vehicles, in which wired communications are not feasible.

In particular, when random data loss affects the platoon dynamics, a stochastic behaviour of the agents response is unavoidable. To the best of our knowledge, only a few authors have focused their attention on string stability in a stochastic framework [11], [12]. In [11], an extension of the definition of string stability from [5] has been given for a stochastic setting. However, this definition is in continuous-time. An alternative definition was also introduced in [12], where the system response to oscillations is bounded. How these concepts apply for the random dropouts case remains unclear. On the other hand, packet loss has been considered for platooning problems in [13], [14], [15]. However, the random nature of the dropouts is not covered and a stochastic analysis for string stability is not fully included in these works.

In this paper, our goal is to motivate the study of the effect that unreliable communications have over discrete-time multi-agent systems designed to behave *properly* in a deterministic setting. Our main contributions include:

- The computation of the infimal value of a design parameter in order to achieve a string stable platoon of LTI agents described by discrete-time models. This is an extension of the results presented in [10].
- A simulation-based study for the effect that lossy communications have in the aforementioned control architecture. The study suggests that the design parameter must increase (compromising performance) in order to maintain a decent beahvior, as the probability of data-loss in the channels increases.

II. Platoon setup over lossy channels

We consider a collection of $N \in \mathbb{N}$ identical agents (commonly vehicles), each modelled by a feedback system composed by a discrete-time LTI plant G, and its local LTI controller C. We assume that the corresponding transfer function of the plant G has at least one integrator and is strictly proper.

The i-th agent, $1 \leq i \leq N$, has access to its own position at the time instant $k \in \mathbb{N}$, denoted by $x_i(k)$, but also to the position of its predecessor $x_{i-1}(k)$, by means of wireless communication channels connecting two consecutive vehicles. Then, the inter-vehicle distance $\ell_i(k) \triangleq x_{i-1}(k) - x_i(k)$ is known by the i-th vehicle whenever the wireless communication works properly (see Fig. 1).

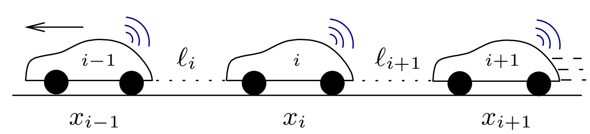

Fig. 1. One dimensional platooning configuration.

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The control objective is to maintain the inter-vehicle distances $\ell_i(k)$ equal to a desire reference $r_i(k)$ whenever possible. This would imply that the agents are either moving together with a constant speed or are stationary, in a desired formation. We will assume that the transfer function of the controller is proper and has integral action.

The wireless communication channel of each vehicle is assumed to be affected by random data-loss, modelled by a Bernoulli process θ_i, such that $\theta_i(k) = 1$ when the predecessor position $x_{i-1}(k)$ is received successfully by the i-th vehicle. If the data is lost, $\theta_i(k) = 0$. We assume that θ_i, $i = 1, 2, \ldots N$, are mutually independent i.i.d. processes, each with the same probabilities of successful transmission p, and failure $1 - p$.

When $x_{i-1}(k)$ is lost, the i-th vehicle cannot determine the current error $e_i(k) \triangleq \ell_i(k) - r_i(k)$. We consider a local strategy that maintains the current speed whenever a data is lost. This is equivalent to set the controller input $e_i(k) = 0$. Hence, the corresponding control scheme in each vehicle is depicted in Fig. 2, where $u_i(k)$ is the control signal and $d_i(k)$ is an input disturbance.

In order to decrease the collision probability, we will assume that as the i-th vehicle speed increases, so does $r_i(k)$. We consider

$$r_i(k) = \varepsilon_i + h(x_i(k) - x_i(k-1)),$$

where $\varepsilon_i > 0$ is a constant value that represents a minimum desired inter-vehicle spacing, the difference $x_i(k) - x_i(k-1)$ gives information about the vehicle speed,[1] and $h \geq 0$ is a constant that weights the importance of vehicle speed.

***Remark** 1:* The constant h is called *time headway constant*. For $h = 0$ we have a constant spacing policy. The control aims to maintain a fixed distance ε_i between the agents. If $h > 0$, the control aims to maintain distances between the vehicles that increase with the rate of change of the agent positions, resembling a continuous-time constant time-headway spacing policy [10]. ■

For simplicity in the exposition we will assume that each vehicle length is zero, $\varepsilon_i = 0$, and that h and the initial conditions $x_i(0)$ and $x_i(-1)$ for $i = 1, \ldots, N$ are compatible, that is, they are such that $e_i(0) = 0$ for $i = 1, \ldots, N$. In this way, we only study the effect of the disturbances $d_i(k)$ on the separation errors $e_i(k)$.

Since $\ell_i(k) = x_{i-1}(k) - x_i(k)$, we can write

$$e_i(k) = x_{i-1}(k) - x_i(k) - r_i(k), \tag{1}$$

and thus each local control loop can be viewed as the one in Fig. 3, with $W = (1+h) - hq^{-1}$, where q^{-1} is the backward shift operator.

The main goal of this paper, is to study the effect of the wireless channels on the platoon string stabilisability. Before doing so, we first define the concept of string stability assuming perfect communication, and then we study the effect of random data loss in the channels.

[1] Since the framework is in discrete-time, this difference is not equivalent to the speed of an agent that travels in continuous-time, but it resembles such quantity.

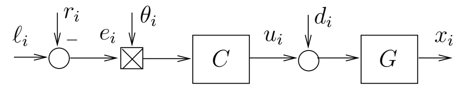

Fig. 2. Control scheme with data-loss.

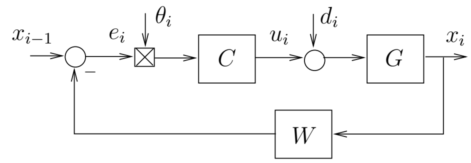

Fig. 3. Equivalent feedback control loop.

III. String stability over lossless channels

We study string stability in the deterministic case. We use capitals to denote the $\mathcal{Z}$-transforms of corresponding time variables. We omit the argument (z) if not needed.

If $p = 1$, $\theta_i(k) = 1$ for all k, i. From Fig. 3, we have that the vehicle dynamics are (with zero initial conditions)

$$\underline{X} = (\boldsymbol{I} - GC\boldsymbol{P})^{-1} G\underline{D}, \tag{2}$$

where $\underline{X} = [X_1 \ \cdots \ X_N]^\top$, $\boldsymbol{I}$ is the $N \times N$ identity matrix, $\underline{D} = [D_1 \ \cdots \ D_N]^\top$ and $\boldsymbol{P} \in \mathbb{C}^{N \times N}$ is given by

$$\boldsymbol{P} = \begin{bmatrix} 0 & & & \\ 1 & -W & & \\ & \ddots & \ddots & \\ & & 1 & -W \end{bmatrix}, \tag{3}$$

with $W = (1+h) - hz^{-1}$.

It is straightforward to show that the effect on the n-th inter-vehicle spacing of a disturbance at the j-th is given by [7]

$$E_n = F_n D_j = \left(\frac{T}{W}\right)^{n-2-j} \mathcal{F} D_j \,, \tag{4}$$

where $\mathcal{F}$ is a transfer function that does not depend on N or j, and T is the modified complementary sensitivity function of the local closed loops

$$T = \frac{GWC}{1 + GWC}. \tag{5}$$

***Definition** 1:* Let $\{F_n\}$ be a sequence of stable transfer functions. The sequence will be called **string stable** if there exists $c \in \mathbb{R}$, independent of $n \in \mathbb{N}$, such that $||F_n||_\infty \leq c$ for all n. It will be called **string unstable** otherwise. ■

Note that this definition of string stability, valid for this setup, ensures that the errors due to disturbances do not amplify when increasing the string size.

The following lemma, taken from [16], implies that the strategy considered above is string unstable whenever $h = 0$.

***Lemma** 1:* Let T be a real rational scalar function of $z \in \mathbb{C}$. Suppose that $T(1) = 1$ and also that T is stable. Then

$$\int_0^\pi \ln \left| T(e^{j\theta}) \right| \frac{d\theta}{1 - \cos(\theta)} \geq \pi T'(1). \qquad ■$$

For T given in (5), it is straightforward to show that $T'(1) = 0$, due to the two integrators of the open loop.

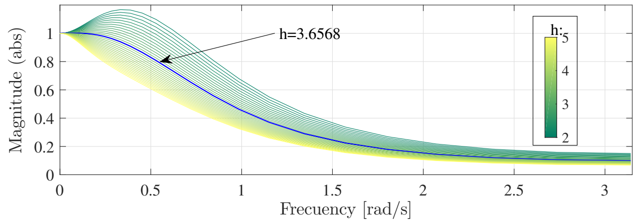

Fig. 4. Bode plots for $|T(e^{j\theta})/W(e^{j\theta})|$ for varying $h \in [2, 5]$

Lemma 1 implies that $||T||_\infty > 1$. If $h = 0$, we have that F_n has an unbounded infinity norm that increases with the number of agents N, and the sequence $\{F_n\}$ is string unstable according to Definition 1. A disturbance having energy in the band where $|T(e^{j\omega})| > 1$, will be amplified along the string.

In order to have string stability, W must be designed accordingly. The work in [10] provides the minimum time headway constant for string stability in continuous-time, whenever the local closed loops are fixed with two integrators in the open loop. We will first aim to recover a similar result for discrete-time.

If we consider controllers that satisfy $C = \tilde{C}/W$, where $\tilde{C}$ does not cancel the dynamics of W, we have that

$$T = \frac{G\tilde{C}}{1 + G\tilde{C}}. \tag{6}$$

By noting that $|W| = \left|e^{j\theta}(1+h) - h\right| = \sqrt{1 + 2h(1+h)(1-\cos\theta)}$, we can conclude that the condition for string stability is that, for all θ,

$$\left|\frac{T}{W}\right| = \frac{1}{\sqrt{1 + 2h(1+h)(1-\cos\theta)}}\left|T(e^{j\theta})\right| \leq 1. \tag{7}$$

Following the argument presented in [10] for the continuous-time case, we have that the infimal value to obtain string stability h_0 is given as the positive root of $2h(1+h) - c = 0$, where $c = \sup_{\theta\in(0,\pi)}\left\{\left(\left|T(e^{j\theta})\right|^2 - 1\right)/(1-\cos\theta)\right\}$.

***Example** 1:* Consider

$$G(z) = \frac{1}{(z-1)}, \quad \tilde{C}(z) = \frac{1.1548(z - 0.7832)}{(z-1)(z+0.8306)}.$$

The corresponding complementary transfer function satisfies $||T||_\infty \approx 1.856$ and $c \approx 29.25$. According to our derivations, the infimal time headway for string stability is $h_0 \approx 3.6568$. This coincides with Fig. 4, where Bode plots for $(|T(e^{j\theta})/W(e^{j\theta})|$ are given for some values of h. Time responses are given in Fig. 5 for two values of h, above and below the limit h_0. For $h = 4$, a string stable behavior is observed. When $h = 2$, the platoon becomes string unstable. This bad behavior is extreme for a larger platoon size, as illustrated in Fig. 5 (for $N = 50$).

IV. String Stability over Lossy Channels

When dropouts occur in the communication channels, the smooth behaviour of Fig 5 cannot be expected. For the same example given in the previous section, particular

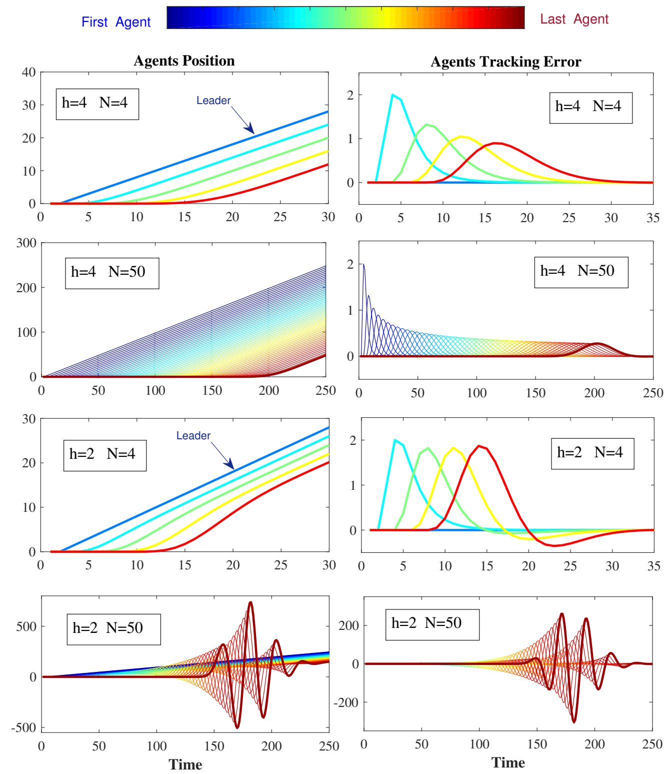

Fig. 5. Vehicle trajectories (left column) and tracking errors (right column) when $h = 4$ and $h = 2$

realizations for the error signals are expected to be like the ones given in Fig. 6, where different probabilities of successful transmission p and constant time headway h are considered. The deterministic string stability concept defined in the previous section is not applicable. However, we can observe similar patterns from the simulation results. For example, Fig. 6-left seems to suggest that the error envelope decreases. This could be considered a good behaviour. On the other hand, Fig. 6-right, shows that the error envelope is not decreasing, and exhibits higher magnitudes and erratic behaviour, before dropping to zero. Recall that, even for string unstable systems, every tracking error converges to zero in this setup, since they follow a constant reference and the controller has integral action. However, we focus on the error magnitudes along the platoon. Fig. 6-centre shows cases in which the behaviour is hard to determine by simple visual inspection. Fig. 6 also suggest that there exists a compromise between h and p, as expected.

If we analyse the mean and variance of the platoon errors, we obtain Fig. 7 for $h = 4.5$. The left column ($p = 0.9$) exhibits an acceptable performance, where no inter-vehicle collisions were detected over 2×10^4 realizations. The right column ($p = 0.6$) shows poor performance, associated to string instability. This suggest that string stability for a stochastic scenario might depend on the mean and variance of the tracking error.

Simulation results for the relation between p and h are given in Fig 8. It can be seen that h must increase when

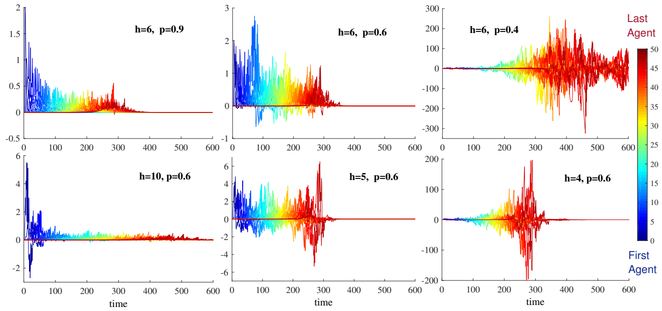

Fig. 6. Vehicles errors for different values of h and p

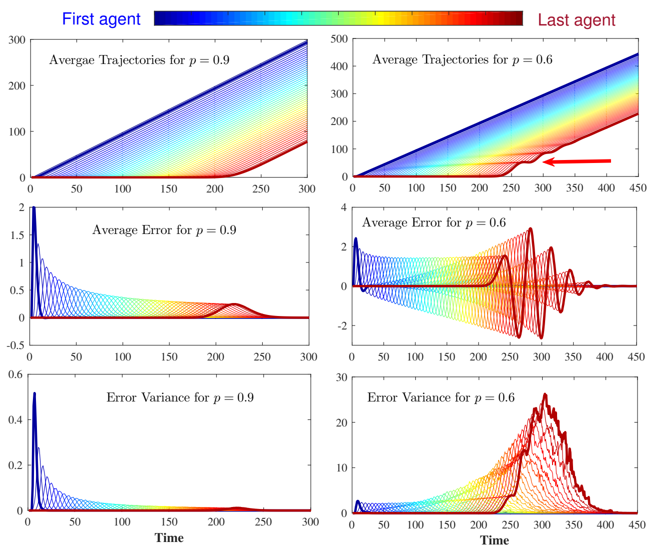

Fig. 7. Sample mean and variance over 2×10^4 realizations when $h = 4.5$ and $N = 50$, for $p = 0.9$ and $p = 0.6$

using poorer quality channels to obtain a stochastic string stable behaviour. The plot also suggest that there exists a minimum value for p beyond which string stability is not possible.

V. Conclusions

We have provided a method to obtain the infimal time-headway constant for string stability of a discrete-time LTI platoon with nearest-neighbour unidirectional topology. We also performed a computational analysis of the impact that data losses have in such architectures. It was noted that when the probability of data loss increases, the behavior of the platoon becomes unacceptable under a simple performance criteria, unless the time headway constant is also increased. We can conclude that data loss can compromise the safety and performance of a platoon that was deemed string stable in a deterministic setting with perfect communications. In order to obtain a clear understanding of the relation between data loss and string stability more work is needed.

We will aim to better understand the current setting, in order to obtain compensation schemes in the communication channels that guarantee minimal impact of data losses over the desired properties of a platoon.

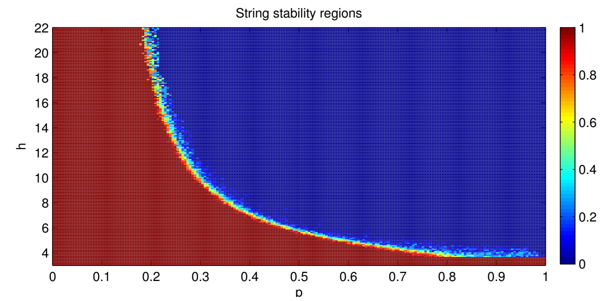

Fig. 8. Approximate regions of the parameter space (h, p) where string stability/instability is observed

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