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Quantitative Analysis of Controller Design Localizability

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***Abstract*— In this paper, we propose an index of the localizability for the distributed design of local controllers. We consider the case where a large-scale system is controlled by a retrofit controller. The retrofit controller is a local plug-in controller such that, rather than an entire system model, only a model of the subsystem of interest is required for the controller design. In addition, the retrofit controller guarantees robust stability in the sense that the control system is stable for any variations of subsystems other than the subsystem of interest, as long as the entire system before retrofit control is stable. The localizability index is defined as the $\mathcal{H}_\infty$ norm of an error system defined based on the isolation of the subsystem of interest from the entire system. The proposed index measures how much the control performance of the subsystem of interest is invariant with respect to the variation of subsystems other than the subsystem of interest. Then, we show that the localizability index is estimated from only the model parameters of the subsystem of interest. Finally, a retrofit controller placement problem for power systems is analyzed by the localizability index.**

I. INTRODUCTION

Decentralized and distributed control are indispensable for large-scale systems. For example, we consider the control problem of power systems with a large number of renewable generators, e.g., photovoltaics (PV). The entire power system is partitioned into multiple local areas. Then, each local controller, which utilizes only local measurement of each area, is implemented to each of the multiple local areas to maintain the demand and supply balance in the whole power system. In most decentralized and distributed control methods [1], [2], [3], the local controller design is based on the premise that the information of the whole system model is available. From a practical viewpoint, not only the implementation of the local controller but also its design should be performed in a distributed manner; each local controller should utilize model parameters of only its own area.

To improve the control performance of the power system under the installation of renewable generators, the paper [4] proposes a control method making use of batteries with large capacity. The batteries cannot be installed to all buses in the power system due to its high cost. From this fact, it is necessary to install the limited number of batteries to appropriate buses in a suitable sense. With this background, many papers [5], [6], [7] address an actuator or controller allocation problem for large-scale systems. In the papers [6], [7], a solution for the problem is provided based on the controllability Gramian of the large-scale system. Most of

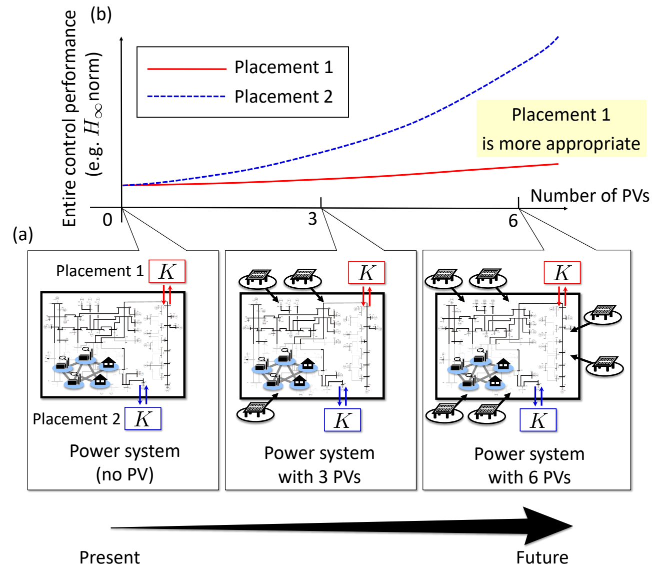

Fig. 1. (a) Controller placement under gradual penetration of PV power systems. (b) Effect of penetration of PV power systems on control performance.

the allocation problems are addressed based on the premise that the composition of the large-scale system never changes, and do not take account into to design and implement the controller in a distributed manner.

For example, we consider the situation in which one controller is applied to a PV-integrated power system, as illustrated in Fig. 1(a). In this situation, PV power systems are assumed to be gradually installed into the power system. The problem to be studied here is to determine a location at which the controller is applied from two possible locations in Fig. 1(a). Figure 1(b) shows the control performance of the power system in Placement 1 and 2, where the origin expresses a present power system without PVs. From Fig. 1(b), we see that the control performance at Placement 2 is the same as that at Placement 1 at the origin, while it becomes worse than that in the Placement 1 as the number of PVs is increased. This is because the model parameters and system structure of the entire power system change due to the gradual penetration of PVs. On the basis of the result in Fig. 1(b), Placement 1 is selected as a more appropriate placement. As shown in the example of Fig. 1, we need to pay attention to not only the control performance at the current situation of the controlled plant, but also the robust performance in the sense that the performance of the entire control system is maintained even if the model parameters and system structure change in a future.

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With this background, this paper aims to provide the quantitative analysis of robust performance in the sense that the control performance is kept invariant to some extent for any variation of the model parameters and system structure. We consider the case where a local controller is implemented to the subsystem of interest in a large-scale system. The local controller is designed by a *retrofit control* method [8], [9], [10]. The retrofit control method enables the distributed design of the local controller; rather than an entire system model, only a model of the subsystem is required for the controller design. In addition, the retrofit controller guarantees the internal stability of the entire control system for any variation of subsystems other than the subsystem of interest, as long as the entire system before retrofit control is stable. Then, we propose an index of the $\mathcal{H}_\infty$ norm of an error system defined based on the isolation of the subsystem of interest from the entire system. In this paper, the proposed index is called by a *localizability index* for the distributed design of the local controller. The localizability index measures how much the control performance of the subsystem of interest is invariant with respect to the variation of subsystems other than the subsystem of interest. It is shown that the localizability index is estimated from only the model parameters of the subsystem of interest. Finally, a retrofit controller placement problem for power systems is analyzed by the localizability index.

Notation: Let M be a complex-valued matrix partitioned as

$$M := \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \in \mathbb{C}^{(p_1+p_2)\times(q_1+q_2)}$$

and let $N_\ell \in \mathbb{C}^{q_2\times p_2}$ and $N_{\rm u} \in \mathbb{C}^{q_1\times p_1}$ be two other matrices. Suppose that there exist $(I_{p_2} - M_{22}N_\ell)^{-1}$ and $(I_{p_1} - M_{11}N_{\rm u})^{-1}$. Then, the lower linear fractional transformation (LFT) with respect to N_ℓ is defined as

$$\mathcal{F}_\ell(M, N_\ell) := M_{11} + M_{12}N_\ell(I_{p_2} - M_{22}N_\ell)^{-1}M_{21}.$$

The upper LFT with respect to $N_{\rm u}$ is defined as

$$\mathcal{F}_u(M, N_{\rm u}) := M_{22} + M_{21}N_{\rm u}(I_{p_1} - M_{11}N_{\rm u})^{-1}M_{12}.$$

The symbol $\mathcal{RH}_\infty$ denotes the set of all proper and real rational stable transfer matrices.

II. Preliminary: System Description and Retrofit Controller Design

A. System Description

We consider an interconnected system $G_{\rm Pre}$ composed of two dynamical systems G and $G_{\rm E}$ as illustrated in Fig. 2. The interconnected system $G_{\rm Pre}$ is assumed to be a stably operated system, which has been already controlled by some preexisting controllers. From this fact, the interconnected system $G_{\rm Pre}$ is called by a preexisting system in this paper. From the preexisting system $G_{\rm Pre}$, we extract the subsystem of interest, which is called by a local system G. On the other hand, a part of subsystems other than the local system is called by an environment $G_{\rm E}$. Towards further improvement of the local control performance, we implement a local controller K, which utilizes a local measurement, to the local system G, as illustrated in Fig. 3.

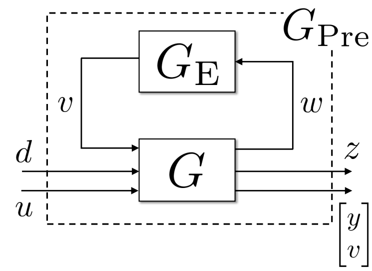

Fig. 2. Preexisting system $G_{\rm Pre}$ composed of a local system G and an environment $G_{\rm E}$

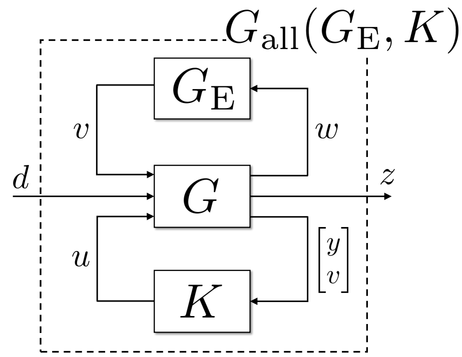

Fig. 3. Control system $G_{\rm all}(G_{\rm E}, K)$

The local system G is described by

$$\begin{bmatrix} w \\ z \\ y \\ v \end{bmatrix} = \begin{bmatrix} G_{wv} & G_{wd} & G_{wu} \\ G_{zv} & G_{zd} & G_{zu} \\ G_{yv} & G_{yd} & G_{yu} \\ I & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ d \\ u \end{bmatrix} := G \begin{bmatrix} v \\ d \\ u \end{bmatrix}, \qquad (1)$$

where $w \in \mathbb{R}^r$ and $v \in \mathbb{R}^s$ denote the interconnection signals from G and $G_{\rm E}$, respectively. The symbols $z \in \mathbb{R}^p$ and $d \in \mathbb{R}^q$ denote the control output and the disturbance input, respectively, and $y \in \mathbb{R}^\ell$ and $u \in \mathbb{R}^m$ denote the measurement output and the control input, respectively. The symbol $G_{\bullet *}$ denotes the transfer function matrix from the signal $*$ to the signal $\bullet$. In the following, we utilize the same notation as the system description G, $G_{\rm E}$, and K for the notation of its transfer function. From (1), we see that the interconnection signal v from $G_{\rm E}$ is included in the output signal of G. This implies that the interconnection signal v is measurable in addition to the signal y. On the other hand, the environment $G_{\rm E}$ is described by

$$v = G_{\rm E}w. \qquad (2)$$

The preexisting system $G_{\rm Pre}$ is described by

$$\begin{bmatrix} z \\ y \\ v \end{bmatrix} = G_{\rm Pre} \begin{bmatrix} w \\ u \end{bmatrix}, \qquad (3)$$

where $G_{\mathrm{Pre}} \in \mathbb{C}^{(\ell+p+s)\times(q+r)}$ is described by

$$G_{\mathrm{Pre}} := \mathcal{F}_{\mathrm{u}}(G, G_{\mathrm{E}}).$$

The local controller K is described by

$$u = K \begin{bmatrix} y \\ v \end{bmatrix}.$$

Then, the control system $G_{\mathrm{all}}(G_{\mathrm{E}}, K)$ in Fig. 3 is described by

$$z = G_{\mathrm{all}}(G_{\mathrm{E}}, K)d,$$

where $G_{\mathrm{all}}(G_{\mathrm{E}}, K) \in \mathbb{C}^{p\times q}$ is described by

$$G_{\mathrm{all}}(G_{\mathrm{E}}, K) := \mathcal{F}_{\ell}(\mathcal{F}_{u}(G, G_{\mathrm{E}}), K).$$

In the following discussion, we assume that both local system and preexisting system are internally stable, i.e., G and $\mathcal{F}_{\mathrm{u}}(G, G_{\mathrm{E}})$ are stable. For simplicity of notation, the set of environment G_{E} that is allowed in this problem setting is denoted as

$$\mathcal{S}_{\mathrm{E}} := \{G_{\mathrm{E}} | \mathcal{F}_{\mathrm{u}}(G, G_{\mathrm{E}}) \in \mathcal{RH}_{\infty}\}.$$

Furthermore, we assume to design the local controller K in a distributed manner; for the local controller design, only the model parameters of G are assumed to be known, while those of the environment G_{E} are unknown.

B. Retrofit Controller

Most of conventional robust control problems [11] impose some norm-bounded condition with respect to the environment G_{E} (the uncertainty part), e.g., for some constant μ, $\|G_{\mathrm{E}}\|_{\infty} \le \mu$ holds. The problem setting in this paper imposes the internal stability of the preexisting system G_{Pre}, while it does not explicitly impose the norm-bounded condition with respect to G_{E}. This premise of new robust control finds out a specific structure in the local controller, which reflects only the model parameter of the local system. This implies that the local contrller is designable in a distributed manner. Such a special local controller is called by a *retrofit controller* [8], [9], [10]. The retrofit controller K enables to guarantee the internal stability of the control system $G_{\mathrm{all}}(G_{\mathrm{E}}, K)$ for any $G_{\mathrm{E}} \in \mathcal{S}_{\mathrm{E}}$.

Let us consider the local controller given by

$$K := \hat{K} \begin{bmatrix} I_{\ell} & -G_{yv} \end{bmatrix}, \tag{4}$$

where $\hat{K} \in \mathbb{C}^{m\times\ell}$ is the transfer function such that $\hat{K}(I_{\ell} - G_{yu}\hat{K})^{-1} \in \mathcal{RH}_{\infty}$ holds, i.e., $\hat{K}$ is a stabilizing controller for the local system G isolated from the environment G_{E}. For simplicity of notation, we denote the set of all stabilizing controllers $\hat{K}$ as

$$\mathcal{S}_{\hat{K}} := \{\hat{K} | \hat{K}(I_{\ell} - G_{yu}\hat{K})^{-1} \in \mathcal{RH}_{\infty}\}.$$

In the following proposition, we provide the stability of the control system with the local controller in the form of (4).

Proposition 1: Suppose that G is internally stable and G_{wu} is left invertible. Then, the control system $G_{\mathrm{all}}(G_{\mathrm{E}}, K)$ is internally stable for any $G_{\mathrm{E}} \in \mathcal{S}_{\mathrm{E}}$ if and only if K has the form of (4) with $\hat{K} \in \mathcal{S}_{\hat{K}}$.

The proof of Proposition 1 is omitted in this paper; see e.g., [9], [10].

Proposition 1 characterizes the class of all local controllers such that the control system $G_{\mathrm{all}}(G_{\mathrm{E}}, K)$ is internally stable for any $G_{\mathrm{E}} \in \mathcal{S}_{\mathrm{E}}$. The local controller given by the form of (4) is called by a *retrofit controller* [8], [9], [10]. From (4), the retrofit controller consists of a part G_{yv} of the model parameter G and the controller $\hat{K} \in \mathcal{S}_{\hat{K}}$ that stabilizes the local system G_{yu}. This implies that the retrofit controller design requires only the model parameter of G, not that of G_{E}. The retrofit controller given by (4) is a simplified version. More general form of the retrofit control without the measurement of the interconnection signal v is also proposed in [8], [9], [10].

III. Quantitative Analysis of Localizability for Retrofit Controller Design

As stated in Proposition 1, the retrofit controller enables to guarantee the internal stability of the control system $G_{\mathrm{all}}(G_{\mathrm{E}}, K)$ for any $G_{\mathrm{E}} \in \mathcal{S}_{\mathrm{E}}$. In the control performance, it is desirable to have high robustness in the sense that the local control performance $\|G_{\mathrm{all}}(G, K)\|_{\infty}$ is kept invariant for any variation in the environment G_{E}. In this paper, such robustness is called by the *localizability* for the distributed design of the local controller. In this section, confining our attention of the local controller to the retrofit controller, we give the quantitative analysis of the localizability for the retrofit controller design.

To define such a quantitative index, we introduce an error system between $G_{\mathrm{all}}(G_{\mathrm{E}}, K)$ and $G_{\mathrm{all}}(0, K)$, the latter of which corresponds to the control system without the environment G_{E}. Then, the error system is simply given by

$$\Delta(G_{\mathrm{E}}, K) := G_{\mathrm{all}}(G_{\mathrm{E}}, K) - G_{\mathrm{all}}(0, K). \tag{5}$$

Definition 1: Consider the control system $G_{\mathrm{all}}(G_{\mathrm{E}}, K)$ with the retrofit controller (4). Then,

$$f(G_{\mathrm{E}}, K) := \|\Delta(G_{\mathrm{E}}, K)\|_{\infty}$$

is said to be a *localizability index* for the distributed design of the local controller.

The localizability index $f(G_{\mathrm{E}}, K)$ is defined as the $\mathcal{H}_{\infty}$ norm of the error system $\Delta(G_{\mathrm{E}}, K)$. To give an interpretation of $f(G_{\mathrm{E}}, K)$, we consider $f(G_{\mathrm{E}}, K) = 0$. Then, it follows that $\|G_{\mathrm{all}}(G_{\mathrm{E}}, K)\|_{\infty} = \|G_{\mathrm{all}}(0, K)\|_{\infty}$. This implies that the local control performance $\|G_{\mathrm{all}}(G_{\mathrm{E}}, K)\|_{\infty}$ is invariant for any variation in the environment G_{E}. From this fact, we see that the value of $f(G_{\mathrm{E}}, K)$ assesses the influence of the environment G_{E} to the local control performance $\|G_{\mathrm{all}}(G_{\mathrm{E}}, K)\|_{\infty}$.

In the following discussion, we state how to calculate the localizability index $f(G_{\mathrm{E}}, K)$. Note that it is impossible to directly calculate $f(G_{\mathrm{E}}, K)$ because the model parameters of the environment G_{E} are assumed to be unknown. Instead of the direct calculation, we develop an estimation method of $f(G_{\rm E}, K)$ based on its upper or lower bound. To this end, we first provide the upper bound of $f(G_{\rm E}, K)$ in the following theorem.

Theorem 1: Consider the control system $G_{\rm all}(G_{\rm E}, K)$ with the retrofit controller (4). Then, for any $G_{\rm E} \in \mathcal{S}_{\rm E}$, it follows that

$$f(G_{\rm E}, K) \leq \alpha \varepsilon(\hat{K}), \tag{6}$$

where nonnegative constants are given by

$$\alpha := \|G_{zv} G_{\rm E} (I_r - G_{wv} G_{\rm E})^{-1}\|_\infty,$$
$$\varepsilon(\hat{K}) := \|\mathcal{F}_\ell(\Pi, \hat{K})\|_\infty,$$

and the transfer function $\Pi \in \mathbb{C}^{(r+\ell)\times(q+m)}$ is described by

$$\Pi := \begin{bmatrix} G_{wd} & G_{wu} \\ G_{yd} & G_{yu} \end{bmatrix}. \tag{7}$$

The proof of Theorem 1 is omitted in this paper.

As stated in Theorem 1, the localizability index $f(G_{\rm E}, K)$ is bounded by $\alpha\varepsilon(\hat{K})$, which is the product of the unknown parameter α and the known parameter $\varepsilon(\hat{K})$. Then, $\varepsilon(\hat{K})$ is described by the $\mathcal{H}_\infty$ norm of the feedback system composed of

$$\begin{bmatrix} w \\ y \end{bmatrix} = \Pi \begin{bmatrix} d \\ u \end{bmatrix}$$

and $u = \hat{K}y$. This implies that the value of $\varepsilon(\hat{K})$ assesses the control performance from the disturbance d injected into G to the interconnection signal w.

Remark 1: We consider the minimization of $f(G_{\rm E}, K)$ with respect to $\hat{K} \in \mathcal{S}_K$. From (6), the minimum value of $f(G_{\rm E}, K)$ is bounded by the product of α and the minimum value of $\varepsilon(\hat{K})$. Note that the minimization problem of $\varepsilon(\hat{K})$ is reduced to that of the $\mathcal{H}_\infty$ norm of the feedback system $\mathcal{F}_\ell(\Pi, \hat{K})$. Then, the minimization problem corresponds to the design problem of the optimal $\mathcal{H}_\infty$ controller for Π. This implies that, in the sense of the upper bound, the localizability $f(G_{\rm E}, K)$ can be estimated from the $\mathcal{H}_\infty$ norm of the feedback system $\mathcal{F}_\ell(\Pi, \hat{K})$ with the optimal $\mathcal{H}_\infty$ controller $\hat{K}$.

Remark 2: The lower bound of the localizability index $f(G_{\rm E}, K)$ can be estimated from the measurement data in an actually operated system. We consider the case where the retrofit controller is implemented to the local system G, and the control system is actually operated. In addition, suppose that some disturbance signal d is injected into the control system $G_{\rm all}(G_{\rm E}, K)$. Then, the response of $G_{\rm all}(G_{\rm E}, K)$, denoted by z_1, can be measurable through the experiment. On the other hand, the response of $G_{\rm all}(0, K)$, denoted by z_2, is obtained through the numerical simulation if the actual disturbance d is realized. This is because the model parameters of G is known. Noting that the output of $\Delta(G_{\rm E}, K)$ is given by $\hat{z} := z_1 - z_2$, we obtain the response of $\Delta(G_{\rm E}, K)$ for the disturbance d. In the sense of the lower bound, the localizability index $f(G_{\rm E}, K)$ can be estimated by combining the experiment with the numerical simulation.

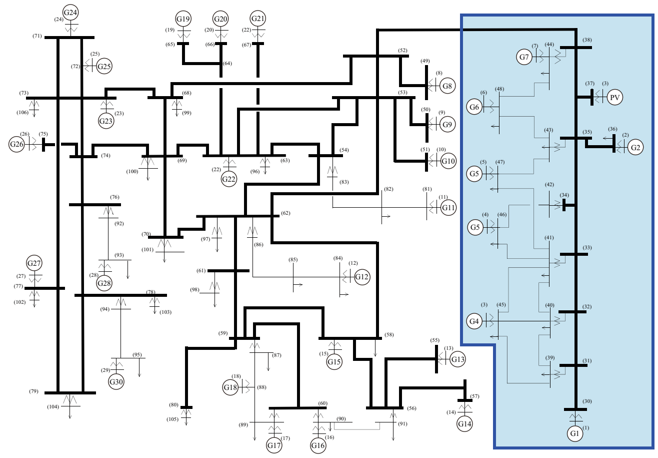

Fig. 4. IEEJ EAST 30-machine model in which one generator is replaced with one PV plant. In the model, the circles and arrows represent synchronous generators and loads, repectively, and the bold lines are the buses. In this case, the local system is illustrated in the blue colored region.

IV. Application to Placement of Retrofit Controller in Power Systems

In this section, the placement problem of the retrofit controller is addressed through an example of power systems. Then, the result is analyzed by the localizability index.

A. Power System Model

As illustrated in Fig. 4, the power system model is based on the IEEE EAST 30-machine power model in which one generator is replaced with one PV plant. The IEEE EAST 30-machine power model [12] represents the power system in the eastern half of Japan. The power system model is composed of 29 generators, 31 loads, 108 buses, and one PV plant. The local system, which is illustrated in the blue colored region in Fig. 4, includes 7 generators and one PV plant. The local system G linearized around an equilibrium point is described by

$$G : \begin{cases} \dot{x} = Ax + Lv + Bu + B_d d, \\ w = \mathit{\Gamma} x + \Xi v, \\ z = Sx, \\ y = Cx + Dv, \end{cases} \tag{8}$$

where $x \in \mathbb{R}^{91}$ is the state of generators in the local system. Each generator has the state with 13 dimensions, which includes the rotor angle, frequency deviation, magnetic flux of an excitation system, and so on. The symbol $v \in \mathbb{R}$ denotes the voltage at the bus in the environment which is connected to G. Let $u := [\,u_1\ u_2\ \ldots\ u_7\,]^\top \in \mathbb{R}^7$ and $y := [\,y_1\ y_2\ \ldots\ y_7\,]^\top \in \mathbb{R}^7$. Then, u_i and $y_i \in \mathbb{R}, i \in \{1, 2, \ldots, 7\}$ denote the input injected into the excitation system and the frequency deviation in each generator, respectively. The symbol $z \in \mathbb{R}$ denotes the average of all frequency deviations in G. In addition, the symbol $d \in \mathbb{R}$ denotes the disturbance and is assumed to be injected into all generators in G.

TABLE I
LOCALIZABILITY INDEX AND CONTROLLABILITY GRAMIAN FOR EACH CONTROLLER PORT SELECTION.

i	$\min_{\hat{K}} \varepsilon_i(\hat{K})$	W_i
1	◯12.8	242.8
2	21.7	192.2
3	20.2	◯262.4
4	15.0	194.6
5	18.3	150.8
6	31.5	159.2
7	44.7	169.2

B. Problem Setting and Solution for Retrofit Controller Placement

Let us implement one retrofit controller to only the ith generator. Then, (8) is rewritten as

$$G: \begin{cases} \dot{x} = Ax + Lv + BE_i u + B_d d, \\ w = \Gamma x + \Xi v, \\ z = Sx, \\ y = E_i C x + Dv, \end{cases} \tag{9}$$

where the matrix $E_i := e_i e_i^\top \in \mathbb{R}^{7\times 7}$ represents the selection of the ith generator to which the retrofit controller is implemented. The stabilizing controller $\hat{K}$ in the retrofit controller is designed as the $\mathcal{H}_\infty$ controller that minimizes the $\mathcal{H}_\infty$ norm of the system with the input d and the output w.

The aim of this section is to provide the placement of the retrofit controller that minimizes the localizability of the control system. In this paper, the localizability means the robustness in the sense that the control performance with respect to the frequency deviation average in the local system is kept invariant to some extent for any variation of the model parameters of the generators in the power system excluding the local system. To provide a solution for the controller placement problem, we utilize the localizability index $f(G_{\rm E}, K)$, in particular, the value of $\varepsilon(\hat{K})$. Let us consider the transfer function $\Pi_i \in \mathbb{C}^{8\times 14}$ described by

$$\Pi_i := \begin{bmatrix} \Gamma(sI - A)^{-1}B_d & \Gamma(sI - A)^{-1}BE_i \\ E_iC(sI - A)^{-1}B_d & E_iC(sI - A)^{-1}BE_i \end{bmatrix}.$$

Then, the optimal controller placement $i_{\rm op}$ is given by

$$i_{\rm op} = \arg\min_i \min_{\hat{K}} \varepsilon_i(\hat{K}), \text{ s.t. } \hat{K} \in \mathcal{S}_{\hat{K}},$$

where $\varepsilon_i(\hat{K}) \in \mathbb{R}$ is given by

$$\varepsilon_i(\hat{K}) = \|\mathcal{F}_\ell(\Pi_i, \hat{K})\|_\infty.$$

C. Numerical Simulation

Table I illustrates the value of $\min_{\hat{K}} \varepsilon_i(\hat{K})$ in the case that the retrofit controller is implemented to the ith generator. To compare the controller placement method based on the controllability Gramian, Table I also illustrates the trace of the controllability Gramian, which is defined as

$$W_i := \int_0^\infty e^{A\tau} BE_iE_iB^\top e^{A^\top \tau} d\tau.$$

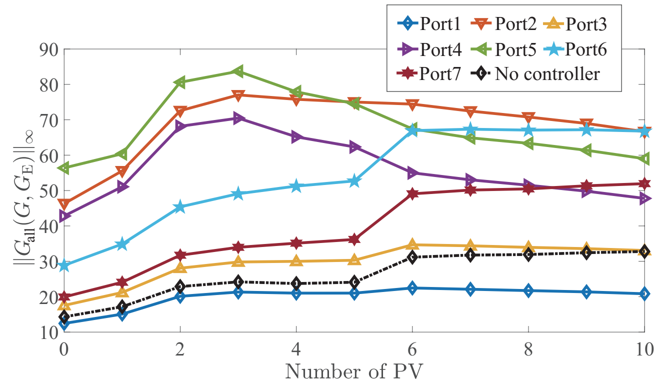

Fig. 5. $\mathcal{H}_\infty$ norm of the control system $G_{\rm all}(G_{\rm E}, K)$ versus the number of PVs.

From Table I, we see that it is optimal to implement to the 1st generator in the sense of the localizability index. On the other hand, in the sense of the controllability Gramian, it is optimal to implement to the 3rd generator. Figure 5 shows the $\mathcal{H}_\infty$ norm of the control system $G_{\rm all}(G_{\rm E}, K)$, where the variation of $G_{\rm E}$ is expressed by replacing generators in $G_{\rm E}$ with PVs. From Fig. 5, the variation of the $\mathcal{H}_\infty$ norm of $G_{\rm all}(G_{\rm E}, K)$ is minimum for the variation of $G_{\rm E}$ in the case of the implementation to the 1st generator. This fact shows the effectiveness of the controller placement based on the localizability index. Furthermore, we see that the $\mathcal{H}_\infty$ norm of $G_{\rm all}(G_{\rm E}, K)$ is also minimum in the case of the implementation to the 1st generator. This fact indicates that the localizability index is also related to the local control performance.

V. CONCLUSION

In this paper, we provided the quantitative analysis of the localizability for the retrofit controller. We addressed the control system with the retrofit controller, which is a local plug-in controller such that only the model of the local system is required for the controller design. The retrofit controller enabled to guarantee the internal stability of the control system for any variation of the environment, as long as the preexisting system is internally stable. The localizability index was defined as the $\mathcal{H}_\infty$ norm of the error system between the control system with and without the environment. The proposed index measured how much the control performance of the local system is invariant with respect to the variation of the environment. Then, it was shown that the localizability index was estimated from only the model parameters of the local system. Finally, the placement problem of the retrofit controller was analyzed through an example of power systems. Then, the effectiveness of the placement based on the localizability index was shown.

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