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Semi-Global Containment Control of Discrete-Time Linear Systems with Actuator Position and Rate Saturation

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This paper studies semi-global containment control problem for a multi-agent system. The dynamics of each follower agent in the system is described by a discrete-time linear system in the presence of actuator position and rate saturation. For each follower agent we construct a linear state feedback control law by using low gain approach such that, the states of follower agents will converge to the convex hull formed by the leader agents asymptotically when the communication topology among follower agents is a connected undirected graph and each leader agent is a neighbor of at least one follower agent.

System description:

Consider a multi-agent system consisting of N networked follower agent, labelled as $1, 2, \cdots, N$. The dynamics of each follower agent $i, i = 1, 2, \cdots, N$, is described by a discrete-time linear system in the presence of actuator position and rate saturation,

$$\begin{cases} x_i(t+1) = Ax_i(t) + B\sigma_{\mathrm{p}}(v_i(t)), \\ v_i(t+1) = v_i(t) + \sigma_{\mathrm{r}}((\alpha-1)v_i(t) + u_i(t)), |\alpha| < 1, \end{cases} \quad (1)$$

where $x_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^m$ and $u_i \in \mathbb{R}^m$ are respectively the plant states, actuator state and control input of i-th agent. $\sigma_{\mathrm{p}}(s)$ and $\sigma_{\mathrm{r}}(s)$ are saturation function represent respectively the position and rate saturation satisfies the following definition.

Definition 1: [1] A function $\sigma : \mathbb{R}^m \to \mathbb{R}^m$ is said to be a saturation function if,

- $\sigma(s)$ is decentralized, i.e., $\sigma(s) = [\sigma_1(s_1), \sigma_2(s_2), \cdots, \sigma_m(s_m)]$; and for each $i = 1$ to m,
- σ_1 is continuous;
- σ_i is linear in a neighborhood of the origin and is bounded away from the vertical axis outside this neighborhood. With out loss of generality assume that within this linear neighborhood the slope is unity, i.e.,

$$\begin{cases} \sigma_i s_i = s_i & \text{if} \quad |s_i| \le 0, \\ \Delta \le |\sigma_i(s_i)| \le b|s_i| & \text{if} \quad |s_i| > 0, \end{cases}$$

for some (arbitrarily small) $\Delta > 0$ and some (arbitrarily large) $b \ge 1$.

Besides the N networked follower agents, there also exist M leader agents in the multi-agent system, labelled as $N+1, N+2, \cdots, N+M$. The dynamics of each leader agent is also described by a discrete-time linear system,

$$x_i(t+1) = Ax_i(t), \quad (2)$$

where $x_i \in \mathbb{R}^n$ is the state of leader agent i.

Let the dynamics of each agent satisfies the following assumptions.

Assumption 1: All eigenvalues of A are inside or on the unit circle and the pair (A, B) is stabilizable.

Lemma 1: [1] Let Assumption 1 hold. Then, for any $\varepsilon > 0$, there exists a unique matrix $P(\varepsilon) > 0$ which solves the following algebraic Riccati equation:

$$P = A^{\mathrm{T}}PA + \varepsilon I - A^{\mathrm{T}}PB(B^{\mathrm{T}}P(\varepsilon)B + I)^{-1}B^{\mathrm{T}}P(\varepsilon)A, \quad (3)$$

such that $A - B(B^{\mathrm{T}}P(\varepsilon)B + I)^{-1}B^{\mathrm{T}}P(\varepsilon)A$ is asymptotically stable. Moreover,

1) $\lim_{\varepsilon\to 0} P(\varepsilon) = 0$,
2) There exists an $\varepsilon^* \in (0, 1]$ such that for all $\varepsilon \in (0, \varepsilon^*]$,

$$\|P(\varepsilon)^{\frac{1}{2}}AP(\varepsilon)^{-\frac{1}{2}}\| \le \sqrt{2}, \quad (4)$$

$$\|P(\varepsilon)^{\frac{1}{2}}AAP(\varepsilon)^{-\frac{1}{2}}\| \le 2, \quad (5)$$

Graph theory:

We will use an undirected graph $\mathcal{G}_N = \{\mathcal{V}_N, \mathcal{E}_N\}$ to represent the communication topology among the follower agents. In this graph, $\mathcal{V}_N = \{\nu_1, \nu_2, \cdots, \nu_N\}$ is a finite, nonempty set of nodes, each denoting a follower agent, and $\mathcal{E}_N \in \mathcal{V}_N \times \mathcal{V}_N$ is a set of edges. An edge (ν_i, ν_j) in an undirected graph denotes that ν_i and ν_j having access to the information form each other.

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ be a graph generated by graph $\mathcal{G}_N$ and the leaders. In this graph, $\mathcal{V} = \{\nu_1, \nu_2, \cdots, \nu_{N+M}\}$ is a finite, nonempty set of nodes, each denoting an agent, and $\mathcal{E} = \{(\nu_i, \nu_j) : \nu_i, \nu_j \in \mathcal{V}\}$ is a set of edges. The neighborhood of an agent i is defined as $\mathcal{N}_i = \{j : (\nu_j, \nu_i) \in \mathcal{E}\}$. Notice that the leader agents have no neighbors. Let $\mathcal{A} = [a_{ij}]$ be the adjacency matrix associated with $\mathcal{G}$, where $a_{ij} = 1$ if $(\nu_j, \nu_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$ for all $i = 1, 2, \cdots, N+M$. Let $\mathcal{L} = [l_{ij}]$ be the Laplacian matrix associated with $\mathcal{A}$, where $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$. Because the leaders have no neighbors, therefore, the Laplacian matrix $\mathcal{L}$ can be written as the following block matrix

$$\mathcal{L} = \begin{bmatrix} L_N & L_M \\ 0_{M\times N} & 0_{M\times M} \end{bmatrix},$$

where $L_N \in \mathbb{R}^{N\times N}$ and $L_M \in \mathbb{R}^{N\times M}$.

The communication topology considered in this paper satisfies the following assumption.

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Assumption 2: The undirected graph $\mathcal{G}_N$ is connected and each leader agent is a neighbor of at least one follower agent.

Lemma 2: [2]Under Assumption 2, L_N is positive definite, each entry of $-L_N^{-1}L_M$ is nonnegative, and each row of $-L_N^{-1}L_M$ has a sum equal to one.

Problem statement:

Considering a multi-agent system consisting of the group of follower agents (1) and leader agents (2), we try to solve the following problem.

Problem 1: For *a priori* given bounded sets $\mathcal{X}_0 \subset \mathbb{R}^n$ and $\mathcal{V}_0 \subset \mathbb{R}^m$, construct a linear state feedback control law

$$u_i(t) = h_i(x_i(t), v_i(t), \{x_j(t), j \in \mathcal{N}_i\})$$

for each follower agent, which only uses local information, such that all these state feedback control laws together solve the semi-global containment control problem for the follower agents (1) and the leader agents (2). That is, foll all $i = 1, 2, \cdots, N$, there exists a set of nonnegative constants $\alpha_{ij}, j = N+1, N+2, \cdots, N+M$, satisfying

$$\sum_{j=N+1}^{N+M} \alpha_{ij} = 1,$$

such that

$$\lim_{t\to\infty} \left(x_i(t) - \sum_{j=N+1}^{N+M} \alpha_{ij} x_j(t) \right) = 0,\ i = 1, 2, \cdots, N,$$

holds for all $x_i(0) \in \mathcal{X}_0, i = 1, 2, \cdots, N+M$, and $v_i(0) \in \mathcal{V}_0, i = 1, 2, \cdots, N$.

Main results:

We propose a linear state feedback containment control law for each follower agent (1),

$$\begin{aligned} u_i(t) =& -\gamma HA \sum_{j=1}^{N+M} a_{ij}(x_i(t) - x_j(t)) \\ &+ \left(1 - \frac{1}{b}\right) \gamma H \sum_{j=1}^{N+M} a_{ij}(x_i(t) - x_i(t)) \\ &- \left(\alpha + \frac{1}{b} - 1\right) v_i(t) - \gamma HB \sum_{j=1}^{N} (v_i(t) - v_j(t)), \end{aligned} \tag{6}$$

where a_{ij} is the $(i,j)^{\text{th}}$ entry of the adjacency matrix $\mathcal{A}$, $\gamma \leq \frac{1}{\lambda_{\max}(L_N)}$ is a constant and

$$H = (I + B^{\mathrm{T}} PB)^{-1} B^{\mathrm{T}} PA, \tag{7}$$

where P is the solution of the ARE (3).

Now we establish the following result on semi-global containment control for multi-agent systems (1)-(2).

Theorem 1: Let Assumptions 1 and 2 hold. Then, the state feedback containment control laws (6) solve semi-global containment control problem for multi-agent systems consiting of follower agents (1) and leader agents (2). That is, for any given bounded sets $\mathcal{X}_0 \subset \mathbb{R}^n$ and $\mathcal{V}_0 \subset \mathbb{R}^m$, there is an $\varepsilon^* > 0$ such that, for any given $\varepsilon \in (0, \varepsilon^*]$ and for all $x_i(0) \in \mathcal{X}_0, i = 1, 2, \cdots, N+M$, and $v_i(0) \in \mathcal{V}_0, i = 1, 2, \cdots, N$,

$$\lim_{t\to\infty} \left(x_i(t) - \sum_{j=N+1}^{N+M} \alpha_{ij} x_j(t) \right) = 0,\ i = 1, 2, \cdots, N,$$

where $\alpha_{ij}, i = 1, 2, \cdots, N, j = N+1, N+2, \cdots, N+M$, are nonnegative constants satisfying

$$\sum_{j=N+1}^{N+M} \alpha_{ij} = 1.$$

Main contribution:

The main contributions of this paper are two-fold. First, this work consider the containment control problem for a multi-agent system in the presence of actuator position and rate saturation, which has not been investigated in existing results on the containment control [2], [3], [4], [5], [6]. Second, the results of the consensus problem subject to the input saturation [6], [7], [8] are generalized to the multiple leaders case.

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