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The singularity sets of a linear matrix inequality in matrix variables

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The talk will be one of 3 coordinated talks (Helton, Klep, Volčič) concerning inequalities and equalities for functions having matrix variables. Matrix multiplication does not commute, so the functions we study are typically noncommutative (nc) polynomials or rational functions. Such mathematics is central to linear systems problems which are specified entirely by a signal flow diagram and L^2 performance specs on signals. The reason being: a system connection law amounts to an algebraic operation on nc quantities, while L^2 performance constraints, through use of quadratic "storage functions", convert to matrix inequalities.

Studies of commutative polynomial inequalities are central to classical mathematics and come under the heading of Real Algebraic Geometry (RAG). Hence linear systems problems cry out for development of a noncommutative RAG. This field has developed greatly over the last decade, pushed partly by systems motivation and greatly by unfolding the parallels to classical RAG.

1) There are now versions of the classical real algebraic geometry description of when one polynomial p is nonnegative on the domain where another polynomial q is nonnegative; nc Positivstellensätze.
2) Nullstellensätze have been more illusive, which is sad because the Holy Grail has been to find a mixture of Positiv- and Nullstellensätze powerful enough to represent a defining nc polynomial for a set, usefully. Such defining polynomials are typically how a spec is given by a signal flow diagram. Recently, there has been serious progress on Nullstellensätze and the attendant Positivstellensätze driven by systems realization theory.
3) Multivariable nc state space representations underlie much recent progress, including that in (2). On the other hand, they are also used for characterization of stable nc poylnomials and rational functions, which relates to the classical problem of existence of determinantal representations for commutative stable polynomials.
4) A focus of much attention is the inequalities corresponding to convexity.
   a) In the nc world, convex sets defined by polynomial specs are all representable as a Linear Matrix Inequality (LMI).
   b) Since systems problems seldom produce an LMI directly, it is important to have a theory for changing variables to produce an LMI or a theory of convex hulls. While this is hopeless for classical polynomials in commuting variables, there is some theory emerging for matrix variables.
5) Other directions have advanced and may be mentioned.

Most of the work we shall discuss in these talks is done jointly by Meric Augat, Eric Evert, J. William Helton, Igor Klep, Scott A. McCullough, and Jurij Volčič.

No paper will be submitted to the proceedings.

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