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Mixed guidance policies of mobile agents in the cooperative estimation of spatially distributed processes in hazardous environments

Michael A. Demetriou

Abstract— **The guidance of mobile sensor-agents used in the estimation of spatially distributed processes often neglects the effects of the process itself on the health and reliability of the sensor. When the spatially distributed process negatively impacts the health status and functionality of the sensing devices then the standard gradient-based sensor guidance will accelerate the demise of the sensor and negatively impact the performance of the estimator. A mixed policy that takes into account the cumulative effects of the environment on the health status of the sensing devices will provide a compromise between sensor functionality and estimator performance. This work considers a mixed policy where initially an information-sensitive guidance policy is implemented. Using performance-based criteria, the sensor guidance switches to an information-neutral policy and when certain thresholds pertaining to the life expectancy of the sensing devices are exceeded, then the guidance policy switches to an information-averse policy. The proposed mixed-guidance policy is demonstrated with a 2D advection-diffusion partial differential equation.**

Index Terms— **Distributed parameter systems; distributed estimation; mobile sensor network; sensor guidance policy.**

AMS subject classifications. 35K57, 49N45, 49J20.

I. Introduction

While there has been a renewed interest in the use of mobile sensors (sensors on mobile platforms) in the state estimation of spatially distributed processes, little work considered the effects of the environment on the health status of the sensing devices themselves. Representative works on the use of mobile or scanning sensors for state reconstruction and parameter identification of spatially distributed processes are [1], [2], [3], [4], [5] and the references therein.

The use of mobile sensor network for source localization or state reconstruction in toxic or hazardous or threat environments has been considered in [6], [7], [8], [9], [10], [11], [12]. The meager work on the effects of the dynamic process (spatiotemporally varying process) on the sensing devices onboard mobile platforms (mobile sensor network) has been essentially addressed by the author.

In the earlier work [13], [14], the guidance of the mobile sensors was modified in order to account for the effects of accumulated measurements, that is, the past histories of the measured signals. It was assumed that a given sensor can only tolerate an a priori defined level of accumulated measurements. Measuring above the maximum level would render the sensing device inoperable thus leading to a dilemma: guide the sensor to spatial regions with "more" useful information, thereby enhancing the learning properties of the estimator while at the same time accelerating the demise of the sensor, *or* guide the sensor to spatial regions with "less" useful information, thereby attenuating the learning properties of the estimator while at the same time prolonging the life expectancy and operability of the sensor. This necessitated a modification to the standard guidance policy which took on a simple modification: Employ the standard guidance (information-sensitive) whenever the accumulated measurements are below a user-defined threshold and then implement an information-averse guidance the instance the accumulated measurements are above the user-defined threshold and well below the maximum tolerable limit.

Modifications to the guidance policy in [14] allowed for collaborative estimation amongst the mobile sensors, in the sense of coupling their guidance. To clarify the notion of cooperative estimation, we point to two distinct types of collaborative estimation:

- in the first one multiple mobile sensors implement their own decentralized state estimate and through an appropriate consensus protocol, they ensure that their individual state estimates reach a consensus.
- in the second one, multiple mobile sensors collaborate for a centralized state estimator. Their guidance can be coupled to each other either because of stability arguments, such as those presented in [15], [16] which derived the guidance using Lyapunov stability arguments, or because of the modifications presented in [13], [14] which accounted for the collective accumulated measurements of all sensors.

An aspect not considered earlier is the effects of a zero sensor velocity as predicted by the proposed performance-based guidance of the state estimator(s). In the case of a gradient-based guidance, then the sensor velocity can be zero whenever the spatial gradient of the state estimation error at the current sensor position is zero. This implies that the mobile sensor is traversing on a level-set of the state estimation error, or marching along constant values of the state estimator error. Using the modifications in [14] that account for the accumulated measurements, the sensor velocity can also be zero if the accumulated measurements are momentarily equal to the threshold value as set in the mixed guidance that switches from information-sensitive to information-averse. In unsteady processes the above concerns will not necessarily be an issue with a zero velocity as the state estimation error will change in time and space, and the accumulated measurements will keep increasing, thereby becoming greater than the threshold value. The problem comes with the commanded velocities to the platforms carrying

M. A. Demetriou is with Worcester Polytechnic Institute, Aerospace Engineering, Worcester, MA 01609, USA, mdemetri@wpi.edu.

the sensors. Such platforms are not point masses and have mass and inertia meaning they cannot stop on a moment's notice. Even when the commanded velocity is zero, they must continue traversing in the spatial domain.

To remedy the above, a third policy is introduced, termed *information-neutral* guidance policy and is only instituted whenever the measurement error is (near a) constant for a prolonged time or the accumulated measurements are within a small range of the threshold amount set by the user. In this case, the sensor is guided to move long the level set of the state estimation error. Another possibility of the information neutral policy is to have the sensor move along the direction of positive gradient but with a significantly smaller velocity; i.e. the sensor "slows down".

Recapitulating, the information-sensitive guidance is implemented whenever the state estimation error is not constant for a prolonged time period or the accumulated measurements are well-below the chosen threshold value. The sensor motion in this case is normal to the instantaneous level set of the state estimation error and the sensor platform is guided towards the local maximum of the state estimation error. When the accumulated measurements are in the vicinity of the threshold value, then the sensor may be guided towards the instantaneous level set of the state estimation error. Switching between the above two policies is allowed. When the accumulated measurements far exceed the allowable threshold, then the information-averse guidance is implemented and the sensor moves in the direction of the local minimum of the state estimation error. Once the information-averse guidance is activated, switching to either of the other two is not allowed as the information-averse policy signifies that the accumulated measurements have exceeded the threshold and now the sensor policy tries to maximize the life of the sensing device at the possible expense of the performance of the state estimator.

The problem under consideration is presented in Section II along with the sensor-parameterized state estimator. The standard gradient-ascent guidance, first presented in [14] is generalized and is now written explicitly in terms of the gradient vector of the associated state estimation error and presented in Section III. A motivation for the need of a modified guidance (mixed policy) is given in Section IV and the proposed mixed-guidance policy is summarized in Section V. Simulation studies of a 2D diffusion PDE with a single interior sensor implementing the binary guidance policy (information-sensitive to information averse) and the ternary guidance policy (information-sensitive to information-neutral to information-averse) is presented in Section VI. Conclusions follow in Section VII.

II. Problem description

We consider advection-diffusion PDEs in 2 and 3 spatial dimensions. For ease of exposure, this work will focus on the 2D case, but the presented results can easily be extended to the 3D case. In particular, we consider the 2D diffusion PDE with controls at the interior and the boundaries, depicted in

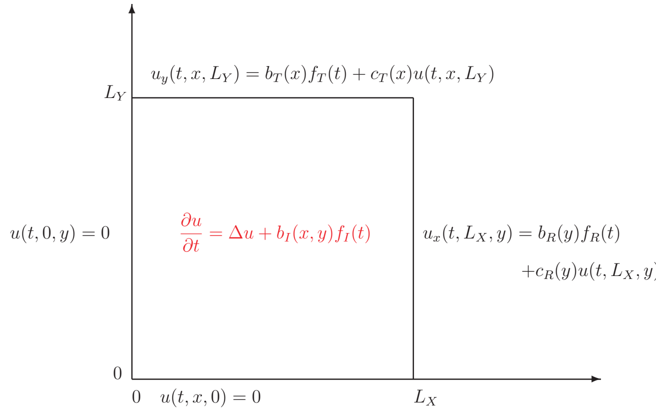


Fig. 1. 2D diffusion PDE with in-domain and boundary controls and observations.

Figure 1 and described by

$$\begin{aligned}
&\frac{\partial u(t,x,y)}{\partial t} = \Delta u(t,x,y) + b_I(x,y)f_I(t)\\
&u(0,x,y) = u_0(x,y) \qquad \text{in } [0,L_X]\times[0,L_Y],\\
&u(t,0,y) = 0, \quad 0 \le y \le L_Y,\\
&u(t,x,0) = 0, \quad 0 \le x \le L_X,\\
&u_y(t,x,L_Y) = b_T(x)f_T(t) + c_T(x)u(t,x,L_Y),\\
&u_x(t,L_X,y) = b_R(y)f_R(t) + c_R(y)u(t,L_X,y).
\end{aligned} \tag{1}$$

The functions $b_T(x)$ and $b_R(y)$ denote the spatial distributions of the actuators at the top and right boundaries, respectively. The control functions $f_T(t)$ and $f_R(t)$ denote the associated control signals. Similarly for the spatial distribution of the interior actuator $b_I(x,y)$ and its associated interior control $f_I(t)$. Associated with the state equation (1) are the measurements provided by both interior and boundary sensors

$$Z(t) = \begin{bmatrix} Z_I(t) \\ Z_B(t) \end{bmatrix} = \begin{bmatrix} \int_\Omega c_I(t,\boldsymbol{\xi})u(t,\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi} \\ \int_{\partial\Omega} c_B(t,\boldsymbol{\xi})u(t,\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi} \end{bmatrix}, \tag{2}$$

where the 2D rectangular domain $\Omega = [0,L_X]\times[0,L_Y]$ with $\boldsymbol{\xi} = (\xi_1,\xi_2) = (x,y)$ and the spatial functions $c_I(t,\boldsymbol{\xi})$, $c_B(t,\boldsymbol{\xi})$ denote the sensor models. Mobile sensors, both in the interior and on the boundaries are parameterized by the time varying centroids $\boldsymbol{\xi}_I(t)$ and $\boldsymbol{\xi}_B(t)$, respectively. The sensor models (2) are now given by

$$\begin{aligned}
Z(t;\boldsymbol{\xi}(t)) &= \begin{bmatrix} Z_I(t;\boldsymbol{\xi}_I(t)) \\ Z_B(t;\boldsymbol{\xi}_B(t)) \end{bmatrix}\\
&= \begin{bmatrix} \int_\Omega c_I(t,\boldsymbol{\xi};\boldsymbol{\xi}_I(t))u(t,\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi} \\ \int_{\partial\Omega} c_B(t,\boldsymbol{\xi};\boldsymbol{\xi}_B(t))u(t,\boldsymbol{\xi})\,\mathrm{d}\boldsymbol{\xi} \end{bmatrix}.
\end{aligned} \tag{3}$$

For the case of a single interior sensor, a single boundary sensor at the top and a single boundary sensor at the right boundary, (3) becomes

$$Z(t;\boldsymbol{\xi}(t)) = \begin{bmatrix} Z_I(t;x_I(t),y_I(t)) \\ Z_T(t;x_T(t)) \\ Z_R(t;y_R(t)) \end{bmatrix}$$

$$= \begin{bmatrix} \int_0^{L_X}\int_0^{L_Y} c_I(t,x_I(t),y_I(t))u(t,x,y)\,\mathrm{d}y\,\mathrm{d}x \\ \int_0^{L_X} c_T(t,x_T(t))u(t,x,y)\,\mathrm{d}x \\ \int_0^{L_Y} c_R(t,y_R(t))u(t,x,y)\,\mathrm{d}y \end{bmatrix}.$$

When the sensor models are taken to be the Dirac delta functions, meaning that the sensors provide pointwise measurements, then the above becomes

$$Z(t;\boldsymbol{\xi}(t)) = \begin{bmatrix} u(t;x_I(t),y_I(t)) \\ u(t;x_T(t),L_Y) \\ u(t;L_X,y_R(t)) \end{bmatrix}. \tag{4}$$

A. State estimator with mobile sensors

Following [14], the observer for (1), (4) takes the form of a Luenberger observer with output injection terms parameterized by the centroids $(x_I(t),y_I(t))$ (for interior), $x_T(t)$ (for top boundary) and $y_R(t)$ (for right boundary) and given by

$$\begin{aligned}
\frac{\partial\widehat{u}(t,x,y)}{\partial t} &= \Delta\widehat{u}(t,x,y)+b_I(x,y)f_I(t) \\
&\quad +\lambda_I(t,x,y)\Big(u(t,x_I(t),y_I(t))-\widehat{u}(t,x_I(t),y_I(t))\Big) \\
\widehat{u}(0,x,y) &= \widehat{u}_0(x,y) \qquad \text{in } [0,L_X]\times[0,L_Y], \\
\widehat{u}(t,0,y) &= 0, \qquad 0\le y\le L_Y, \\
\widehat{u}(t,x,0) &= 0, \qquad 0\le x\le L_X, \\
\widehat{u}_y(t,x,L_Y) &= b_T(x)f_T(t)+c_T(x)\widehat{u}(t,x,L_Y) \\
&\quad +\lambda_T(x;x_T(t))\Big(u(t,x_T(t),L_Y)-\widehat{u}(t,x_T(t),L_Y)\Big), \\
\widehat{u}_x(t,L_X,y) &= b_R(y)f_R(t)+c_R(y)\widehat{u}(t,L_X,y) \\
&\quad +\lambda_R(y;y_R(t))\Big(u(t,L_X,y_R(t))-\widehat{u}(t,L_X,y_R(t))\Big).
\end{aligned} \tag{5}$$

In general, the sensor location-dependent kernels can be derived using a Kalman filter design or a Luenberger observer design. In the former, one must consider the solution to a differential Riccati operator equation in order to extract the filter kernels whereas for the latter, the filter kernels are chosen to satisfy certain stability criteria. The structure of this observer is that of the kernels $\lambda_I(t,x,y)$, $\lambda_T(x;x_T(t))$, $\lambda_R(y;y_R(t))$ be chosen equal to constant multiples of the adjoints of the output operators associated with the output functions (spatial delta functions) and thus

$$\begin{aligned}
\lambda_T(x;x_T(t)) &= \kappa_T\,\delta(x-x_I(t)), \\
\lambda_R(y;y_R(t)) &= \kappa_R\,\delta(y-y_R(t)), \\
\lambda_I(x,y;x_I(t),y_I(t)) &= \kappa_I\,\delta(x-x_I(t))\delta(y-y_I(t)),
\end{aligned} \tag{6}$$

where $\kappa_T,\kappa_R,\kappa_I$ are the user-defined positive gains defined in [14], [16]. What essentially remains in the observer design is the guidance of the mobile sensors, presented in Section III.

Central to the derivation of the sensor guidance is the state estimation error $e(t,x,y)=u(t,x,y)-\widehat{u}(t,x,y)$ governed by

$$\begin{aligned}
\frac{\partial e(t,x,y)}{\partial t} &= \Delta e(t,x,y)-\lambda_I(t,x,y)e(t,x_I(t),y_I(t)) \\
e(0,x,y) &= e_0(x,y) \qquad \text{in } [0,L_X]\times[0,L_Y], \\
e(t,0,y) &= 0, \qquad 0\le y\le L_Y, \\
e(t,x,0) &= 0, \qquad 0\le x\le L_X, \\
e_y(t,x,L_Y) &= -\lambda_T(x;x_T(t))e(t,x_T(t),L_Y) \\
e_x(t,L_X,y) &= -\lambda_R(y;y_R(t))e(t,L_X,y_R(t)).
\end{aligned} \tag{7}$$

III. Guidance of mobile sensors

Incorporating the effects of the environment on the sensor life and reliability results in a modified guidance as presented in [14]. The two types of guidance were *information-sensitive* and *information-averse*, and utilized the accumulated measurements as a means to alter the sensor guidance. Following the relevant earlier works [13], [14], define the following functions that provide information on the accumulated measurements:

- the <u>*accumulated mass*</u>,
- the <u>*maximum mass*</u> and
- the <u>*threshold mass*</u>.

Definition 1: [14][*accumulated mass*] The accumulated mass of the i^{th} sensor is the total amount of the measured process state by the sensor up to the current time t via

$$m_i(t) = \int_0^t Z_i(\tau;\boldsymbol{\xi}_i(\tau))\,\mathrm{d}\tau, \quad i=1,\dots,N. \tag{8}$$

It provides information on the cumulative amount of the spatial field (measurement) that the i^{th} sensor has been exposed to.

Definition 2: [14][*maximum mass*] The maximum mass m_i^{max} is defined as the limit of the maximum exposure to the process state beyond of which the i^{th} sensor becomes inoperative; i.e. it may be useless or saturated and, no longer reading, or no longer transmitting readings, or, transmitting incoherent readings.

A way to keep track of the instance the accumulated mass $m_i(t)$ of the i^{th} sensor exceeds its maximum mass m_i^{max} is via the use of the i^{th} sensor *indicator function*.

$$\mathbf{1}_{m_i}(t) = \begin{cases} 1 & \text{if } m_i(t) < m_i^{max} \\ 0 & \text{if } m_i(t) \ge m_i^{max} \end{cases} \qquad i=1,\dots,N. \tag{9}$$

It allows the mobile sensors to come to a stop the instance the accumulated mass exceeds the maximum mass. A retrieval policy may be instituted to return the sensor to a base station and to retrofit it with a healthy sensor.

Definition 3: [14][*threshold mass*] The threshold mass of the i^{th} sensor, denoted by $m_i^{thresh} \le m_i^{max}$ is a *user-defined* threshold that the guidance policy employs to switch from an *information-sensitive* to an *information-averse* motion in order to prolong the life expectancy of the sensor.

A. Gradient ascent guidance based on accumulated exposure

A vehicle (mobile platform) carrying the i^{th} onboard sensor is assumed to be moving with a maximum speed υ_i^{max}

accumulated mass level	guidance
$m_i^{thresh} - m_i(t) > 0$	information-sensitive
$m_i^{thresh} - m_i(t) < 0$	information-averse

TABLE I
CURRENT MEASUREMENT-DEPENDENT GUIDANCE.

in each direction.

A simple gradient-ascent guidance policy as presented in [14] is given by

$$\dot{\xi}_i(t) = \begin{bmatrix} \dot{x}_i(t) \\ \dot{y}_i(t) \\ \dot{z}_i(t) \end{bmatrix} = \begin{bmatrix} \mathrm{sign}\big(e_x(t,x_i(t),y_i(t))\big) \\ \mathrm{sign}\big(e_y(t,x_i(t),y_i(t))\big) \\ \mathrm{sign}\big(e_z(t,x_i(t),y_i(t))\big) \end{bmatrix} \upsilon_i^{max}.$$

A generalization of this, as first presented here, is expressed in terms of the gradient vector and thus

$$\dot{\xi}_i(t) = \vec{N}(t,\xi_i(t))\,\upsilon_i^{max}, \tag{10}$$

where the unit (outward) normal vector is given by

$$\vec{N}(t,\xi_i(t)) = \frac{\nabla e(t,\xi_i(t))}{|\nabla e(t,\xi_i(t))|} \tag{11}$$

and which points in the direction of increasing $e(t,\xi)$ and is perpendicular to the isocontours (level sets) of $e(\cdot,\xi) - \text{constant} = 0$.

Using the modification (10) that generalizes the gradient ascent guidance policy in [13], along with the modification presented there that accounts for the effects of the accumulated measurements, the guidance is now given by

$$\dot{\xi}_i(t) = \mathbf{1}_{m_i}(t)\mathrm{sign}\big(m_i^{thresh} - m_i(t)\big)\vec{N}(t,\xi_i(t))\,\upsilon_i^{max}. \tag{12}$$

The term $\mathrm{sign}\big(m_i^{thresh} - m_i(t)\big)$ switches from an information-sensitive to an information-averse guidance whenever the accumulated mass $m_i(t)$ exceeds a used-defined threshold $m_i^{thresh} \le m_i^{max}$. This is also presented in Table I. The sign function, written in terms of the shifted step function and depicted in Figure 4(a), can be denoted as

$$\begin{aligned} \mathrm{sign}\big(m_i^{thresh} - m_i(t)\big) &= 1 - 2H(m_i(t) - m_i^{thresh}) \\ &\triangleq \sigma(m_i(t)). \end{aligned}$$

The sensor speed can be modified to a time-varying and *measurement-dependent* speed,

$$\boxed{\upsilon_i(t) = \mathbf{1}_{m_i}(t)\left(1 - \frac{m_i(t)}{m_i^{max}}\right)\upsilon_i^{max}, \quad i = 1,\ldots,N.} \tag{13}$$

where υ_i^{max} is the maximum speed of the i^{th} mobile sensor. Using the above definition of the measurement-dependent speed, the above guidance changes to

$$\boxed{\dot{\xi}_i(t) = \sigma(m_i(t))\,\vec{N}(t,\xi_i(t))\,\upsilon_i(t), \quad i = 1,\ldots,N.} \tag{14}$$

With regards to the single interior sensor, single top boundary sensor and single right boundary sensor ($i = I,T,R$), the modified guidance that differentiates between the information-sensitive and the information-averse guidance is given below. In the particular 2D case considered in Figure 1, the guidance may be simplified using

$$\frac{\nabla e(t,\xi_i(t))}{|\nabla e(t,\xi_i(t))|} \longrightarrow \begin{bmatrix} \mathrm{sign}\big(e_x(t,x_i(t),y_i(t))\big) \\ \mathrm{sign}\big(e_y(t,x_i(t),y_i(t))\big) \end{bmatrix}.$$

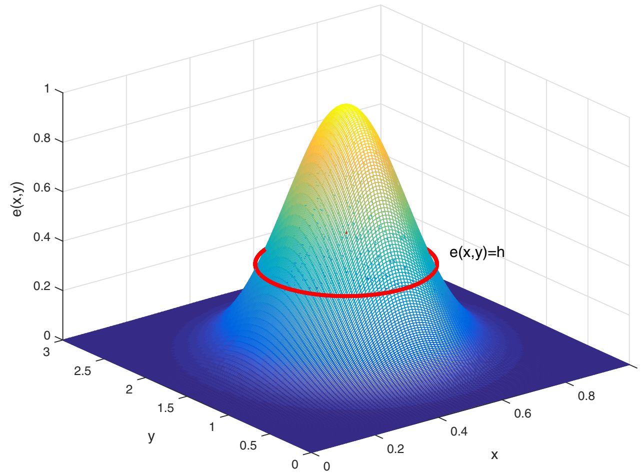


Fig. 2. Spatial distribution of process state and a contour line (level-set) of value $e(x,y) - 0.3247 = 0$.

Thus, for the interior sensor it is

$$\begin{aligned} \dot{x}_I(t) &= \sigma(m_I(t))\,\mathrm{sign}\big(e_x(t,x_I(t),y_I(t))\big)\,\upsilon_I(t) \\ \dot{y}_I(t) &= \sigma(m_I(t))\,\big)\mathrm{sign}\big(e_y(t,x_I(t),y_I(t))\big)\,\upsilon_I(t). \end{aligned}$$

For the top boundary sensor in Figure 1, it is

$$\dot{x}_T(t) = \sigma(m_T(t))\,\mathrm{sign}\big(e_x(t,x_T(t),L_Y)\big)\,\upsilon_T(t),$$

and for the right boundary sensor, it is

$$\dot{y}_R(t) = \sigma(m_R(t))\,\mathrm{sign}\big(e_y(t,L_X,y_R(t))\big)\,\upsilon_R(t).$$

IV. MOTIVATION FOR INFORMATION-NEUTRAL GUIDANCE POLICY

Consider the 2D case at the specific instance where the spatial distribution of the distributed state error $e(x,y)$ is given by the 2D Gaussian distribution

$$e(x,y) = Ae^{-\left[\frac{(x-x_m)^2}{2\sigma_x^2} + \frac{(y-y_m)^2}{2\sigma_y^2}\right]}, \quad A = 1,$$

for $0 \le x \le 1$, $0 \le y \le 3$. In the 1D case, there does not exist a constant value of the spatially distributed state for which a moving agent can have a nonzero velocity. However, in higher dimensions, and in particular in the 2D case considered here, there is a contour line (level-set) with constant values of the state $e(x,y) = h$ for which a moving sensor can move along without coming to a stop. A mobile sensor can move along the contour line given by the ellipse

$$\frac{(x-x_m)^2}{2\sigma_x^2} + \frac{(y-y_m)^2}{2\sigma_y^2} = \log\left(\frac{A}{h}\right),$$

and measuring the same value of the state, i.e. $e(x,y) = h$. While the same value of the process state will be measured, thus adding no information to the state observer, a motion along the level set is applicable to sensors on mobile platforms (e.g. UAVs) that have inertia and cannot hover at a fixed spatial point. However, in such a case, the sensor will still be measuring a non-zero value thus adding to the accumulated mass $m_i(t)$. In this case, the value of the measurements will be exactly $Z(t) = e(x_i,y_i) = h$.

Combining the above, the sensor velocity (12) or (14) can be zero whenever either of the two occur:

- the gradient $\vec{N}(t,\xi_i(t))$ in (12) is zero,
- the accumulated mass $m_i(t)$ is equal to, or near, the threshold mass m_i^{thresh}.

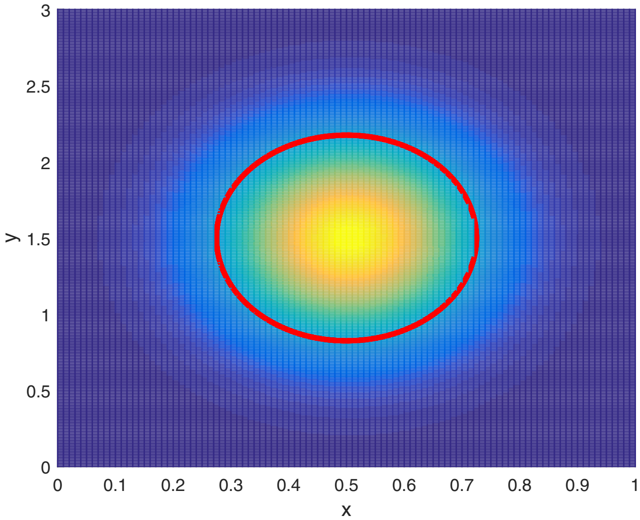


Fig. 3. Contour graph of the process state and the contour line (ellipse) corresponding to a constant value of $e(x,y)-0.3247=0$.

accumulated mass level	guidance
$m_i^{thresh}-m_i(t)>\varepsilon$	information-sensitive
$m_i(t)-m_i^{thresh}>\varepsilon$	information-averse
$\lvert m_i^{thresh}-m_i(t)\rvert\leq\varepsilon$	information-neutral

TABLE II

PROPOSED MEASUREMENT-DEPENDENT GUIDANCE.

The above demonstrates that a third guidance policy can be instituted for the estimation of distributed processes in hazardous environments, that of *information-neutral guidance*. Following Figure 3, an information averse policy would guide the sensor towards the center (yellow region) whereas an information averse would guide the sensor away from the center (blue regions). The third guidance would guide the sensor along the level-set (red ellipse), or towards the yellow region but with a reduced speed.

V. Mixed Guidance Policy

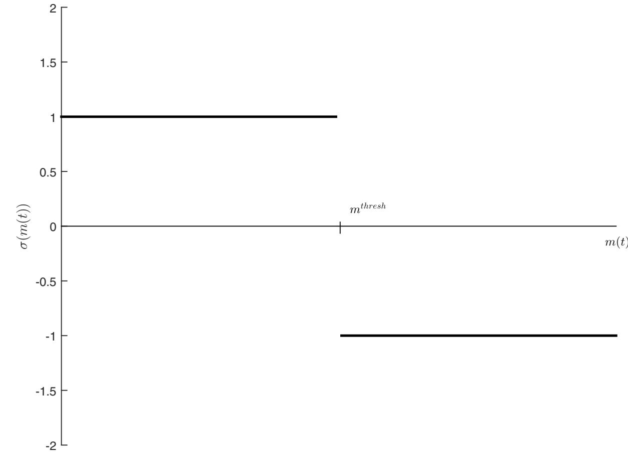
The binary decision for the guidance that switches from an information-sensitive to an information-averse can now be extended to a ternary one that switches from an *information-sensitive* to an *information-neutral* and then to an *information-averse* guidance. The graph of the switching function $\sigma(m_i(t))$ is depicted in Figure 4(b).

While one may resort to a level-set formalism [17], [18] to obtain the mixed guidance policy, we can simply state it below in its simpler form. The binary guidance policy summarized in Table I and given in (11) is now replaced by the ternary policy summarized in Table II and Figure 4(b).

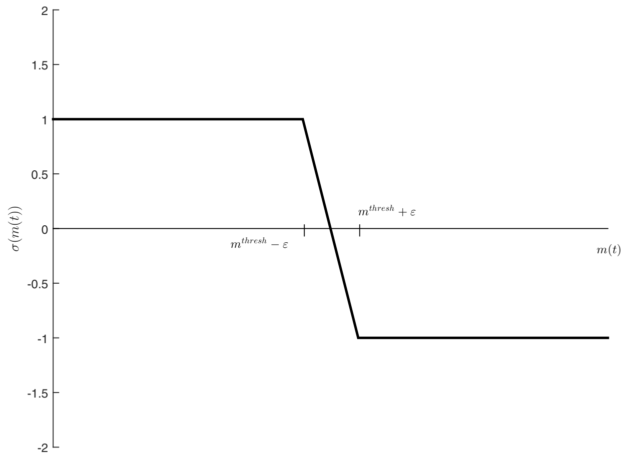
VI. Numerical Studies

Following the earlier work [13], we consider the 2D diffusion equation defined over the spatial domain $[0,L_X]\times[0,L_Y]=[0,1]\times[0,1]$ and given by

$$\frac{\partial u}{\partial t}=\frac{\partial}{\partial x}\big(\alpha(x)\frac{\partial u}{\partial x}\big)+\frac{1}{2}\frac{\partial}{\partial y}\big(\alpha(y)\frac{\partial u}{\partial y}\big)$$
$$+5\frac{\partial u}{\partial x}-6\frac{\partial u}{\partial y}-10^{-2}\kappa u+b(x,y)f_d(t)$$



(a) Binary policy



(b) Ternary policy.

Fig. 4. Current and proposed guidance policies.

where

$$\alpha(x)=0.01\Big[1+0.3\sin(2\pi x)\big(\sin^3(x^3)+\sin^3(L-x)^3\big)\Big],$$
$$\kappa=\big(\frac{\pi}{L_X}\big)^2+\big(\frac{\pi}{L_Y}\big)^2,$$

with initial condition $u(0,x,y)=\sin(\pi x)\big(2+\sin(2\pi y)\big)$ and Dirichlet boundary conditions

$$u(t,0,y)=u(t,L_X,y)=0,\quad 0<y<L_Y,$$
$$u(t,x,0)=u(t,x,L_Y)=0,\quad 0<x<L_X.$$

A finite element approximation scheme using 20 linear splines in each direction, modified to account for the Dirichlet boundary conditions, was used in order to arrive at a finite dimensional approximation. The state estimator initial condition was set to $\widehat{u}(0,x,y)=0$ in Ω having Dirichlet boundary conditions. The filter kernel was set equal to the weighted adjoint of the output operator,

$$\lambda(x,y)=63\delta(x-x_I(t))\delta(y-y_I(t)),$$

with $(x_I(t),y_I(t))$ denoting the coordinates (centroid) of the interior mobile sensor. A single process measurement was given by the interior mobile sensor

$$Z_I(t)=\int_0^{L_X}\int_0^{L_Y}\delta(x-x_I(t))\delta(y-y_I(t))u(t,x,y)\,\mathrm{d}y\,\mathrm{d}x.$$

In addition to the binary guidance presented in (14), the proposed ternary guidance was used with the maximum mass set to $m^{max}=5$ and the threshold mass $m^{thresh}=$

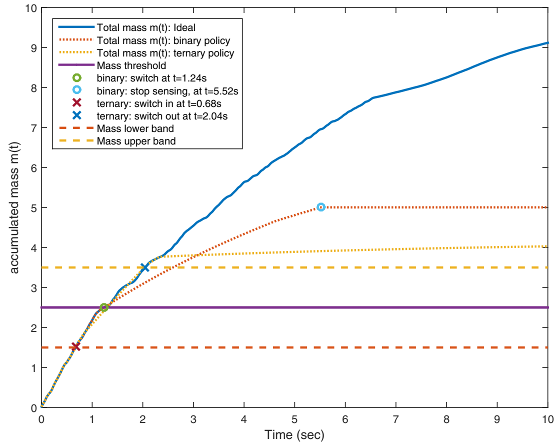


Fig. 5. Evolution of the accumulated mass $m(t)$.

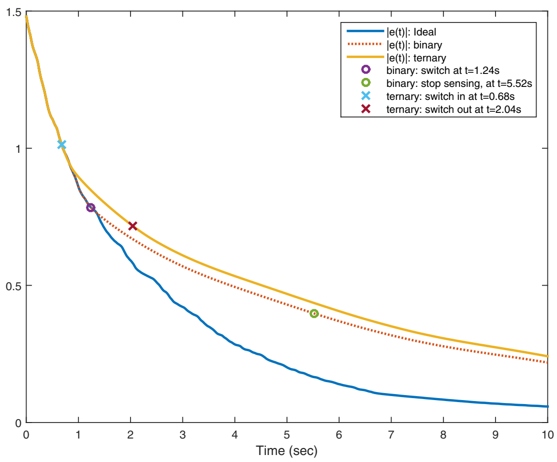


Fig. 6. Evolution of $L_2(\Omega)$ error norm.

$0.5m^{max}$. The banded region where the third guidance policy is implemented is taken to be $\varepsilon = 0.2m^{max}$. which implies that whenever the mass $m(t)$ is between $0.3m^{max} < m(t) < 0.7m^{max}$, the neutral guidance policy is used.

Figure 5 depicts the evolution of the mass $m(t)$. When the binary policy is used, then the sensor switches from information-sensitive to information averse at $t = 1.24$s and exceeds the maximum mass at $t = 5.52$s revealing that it stopped sensing. The ternary policy switches in to the information-neutral policy at $t = 0.68$s and switches out at $t = 2.04$s. The information averse policy is then implemented and the sensor never reaches the maximum mass $m^{max} = 5$ revealing that it still functions beyond the simulation window of $[0, 10]$s. Similar behavior is observed in Figure 6 which depicts the L_2 norm of the state estimation error.

VII. Conclusions

We have considered a modification of the guidance policy for mobile sensors that are deployed in spatial domains and used for the state reconstruction of a spatiotemporally varying field defined over the spatial domain. The guidance policy takes into account the effects of the spatial process on the health status of the sensing device. The negative effects are monitored by the accumulated measurements of the sensing devices. Whenever the accumulated measurements exceed a limit the sensor is taken to be inoperative. The modification to the guidance included a third component, that of information-neutral and which changes the gradient ascent or gradient-descent policy to a neutral third whenever the accumulated mass is within a banded range of the threshold mass. An example of the modified policy for a 2D diffusion process was included to further demonstrate the possible life extension of sensing devices when a mixed (ternary) guidance policy is considered.

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