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Electricity Price Dynamics in the Smart Grid: A Mean-Field-Type Game Perspective

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***Abstract*— In this paper a profit optimization between electricity producers is formulated. The problem is described by a linear jump-diffusion system of conditional mean-field type where the conditioning is with respect to common noise and a quadratic cost functional involving the second moment, the square of the conditional expectation of the control actions of the producers. We provide semi-explicit solution of the corresponding mean-field-type game problem with common noise. The equilibrium strategies are in state-and-conditional mean-field feedback form, where the mean-field term is the conditional price given the realization of the global uncertainty.**

***Index Terms*— Electricity price dynamics, mean-field-type games, smart grid**

I. Introduction

Significant models of competition between producers have been discussed in very simple terms. One widely investigated model is the Cournot model [1]. Evans (1922,[2]) examined the Cournot model for quadratic production cost output. Based on the price model examine in Evans 1922, page 372, Equation (4), the work of Roos 1925 [3], [4] considered demand as depending not only on the present price but on all previous prices as well. This leads to delayed integro-differential equation. In page 163, Roos 1925 [3] defined an open-loop solution concept for deterministic differential Cournot games (which corresponds to the so-called open-loop Nash equilibrium). Simaan and Takayama (1978,[5]) focus on the role of capacity constraints and assume the speed of adjustment to be unity. Fershtman and Kamien (1987,[6]) extended it to allow for an arbitrary adjustment speed. They investigated both open-loop and closed-loop Nash equilibria of a class of deterministic differential Cournot game. Since several interesting studies and body of literature have been conducted on the dynamic Cournot oligopoly problems [10].

We study a stochastic dynamic Cournot game between electricity producers where the price of electricity is adjusted progressively with a local uncertainty, global uncertainty and jump terms. Our base model is similar to Equation (2) page 163 in Ross (1925,[3]). See similar base model in page 372, Equation (4) by Evan (1922,[2]). To that base model we add a key term which is related to the risk minimization [7], [8], [9]. The risk is an important consideration when uncertainty is involved in the environment. In addition, price adjustment is subject to uncertainty which is modeled with a local uncertainty, jump-diffusion and a global uncertainty.

Our goal of this article is to investigate how this simple price model can be used to capture realistic behaviors observed in smart energy systems. Our contribution can be summarized as follows. We formulate a mean-field-type game with common noise and jump-diffusion. The equilibrium systems of such problems often involve a master system which is a second order infinite dimensional partial integro-differential equation. We provide semi-explicit solution to the equilibrium system. We show that, generically, the structure of the optimal strategies is in state-and-mean-field feedback form. Here the key mean-field term is the conditional expectation of the price with respect to the filtration generated by the common noise. Thus, the mean-field is stochastic.

The rest of this paper is organized as follows. The next section introduces the model and the key interaction terms at affect the price. Section III examines longer term horizon. Section IV briefly presents the computation of the Cournot equilibrium when considering a static price. A decision support on the market price is discussed in Section V. Section VI concludes the paper.

II. The Setting

Consider a mean-field-type game described by the following settings. Let $\mathcal{T} := [t_0, t_1]$ be the time horizon with $t_0 < t_1$. Moreover, there are $n \geq 2$ potential interacting energy producers over the horizon $\mathcal{T}$. At time $t \in \mathcal{T}$, producer i's output is $u_i(t) \geq 0$. The price dynamics is given by $p(t_0) = p_0$ and

$$\mathrm{d}p = s[a - D - p]\mathrm{d}t + \left(\sigma \mathrm{d}B + \int_{\theta\in\Theta} \mu(\theta)\tilde{N}(\mathrm{d}t, \mathrm{d}\theta)\right) + \sigma_o \mathrm{d}B_o, \quad (1)$$

where $D(t) := \sum_{i=1}^{n} u_i(t)$ is the supply at time t, and B_o is standard Brownian motion representing a global uncertainty observed by all participant to the market. The processes B and N are local uncertainty or noise. B is a standard Brownian motion, N is a jump process with Lévy measure $\nu(\mathrm{d}\theta)$ defined over Θ. It is assumed that ν is a Radon measure over Θ (the jump space). The process $\tilde{N}(\mathrm{d}t, \mathrm{d}\theta) = N(\mathrm{d}t, \mathrm{d}\theta) - \nu(\mathrm{d}\theta)\mathrm{d}t$ is compensated martingale. We assume that all these processes are mutually independent. Denote by $\mathcal{F}_t^{B_o}$ the filtration generated by the observed common noise up to t, $\{B_o(t'),\ t' \leq t\}$. The number s is positive.

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larger values of s the market price adjusts quicker along the inverse demand. a, σ, σ_o are fixed parameters. The jump rate size $\mu(.)$ is in $L^2_\nu(\Theta, \mathbb{R})$ i.e. $\int_\Theta \mu^2(\theta)\nu(\mathrm{d}\theta) < +\infty$. The initial distribution of p_0 is square integrable: $\mathbb{E}p_0^2 < \infty$. At time $t \in \mathcal{T}$, Producer i receives $\bar{p}(t)u_i - C_i(u_i)$ where $C_i : \mathbb{R} \to \mathbb{R}$, given by

$$C_i(u_i) = c_i u_i + \frac{r_i u_i^2}{2} + \frac{\bar{r}_i \bar{u}_i^2}{2},$$

is the instant cost function of i. The term $\bar{u}_i = \mathbb{E}(u_i \mid \mathcal{F}_t^{B_o})$ is the conditional expectation of producer i's output given the global uncertainty B_o observed in the market, . The last term in the expression of the instant cost C_i, i.e., $\frac{\bar{r}_i \bar{u}_i^2}{2}$, aims to capture the risk-sensitivity of the producer. The conditional expectation of the price given the global uncertainty B_o up to time t is $\bar{p}(t) = \mathbb{E}(p(t) \mid \mathcal{F}_t^{B_o})$. At the terminal time t_1 the revenue is $-\frac{1}{2}e^{-\lambda_i t_1}\left(p(t_1) - \bar{p}(t_1)\right)^2$.

The long-term revenue of producer i is

$$\begin{aligned} R_{i,\mathcal{T}}(p_0, u) = &-\frac{q}{2}e^{-\lambda_i t_1}\left(p(t_1) - \bar{p}(t_1)\right)^2 \\ &+ \int_{t_0}^{t_1} e^{-\lambda_i t}\left[\bar{p}u_i - C_i(u_i)\right]\mathrm{d}t, \end{aligned}$$

where λ_i is a discount factor of producer i. Finally, each producer optimizes its long-term expected revenue.

Moreover, number of remarks are in order.

- This price dynamics model is interesting because it can be re-interpreted as an error to the standard inverse demand model.
- The changing s allow us to navigate between several regime.
- the jump term $\tilde{N}$ capture some of the big change in the market that may happen randomly
- The global uncertainty B_o captures common noise in the market, for example, weather conditions and temperature field in specific season.
- Based on the common noise that is observed, a conditional price is calculated. The revenue is computed from the conditional price.
- This revenue model is similar to the one considered by Jovanovic 1982 (see page 652 in [11]) who studied discrete-time mean-field games for selection and evolution industry. Therein the conditional state appears well as. However, [11] considered that firms are too small to affect the price. Here, each of the n firms can influence the price and cannot be neglected. Global uncertainty was not considered in [11].

A. Why is this a mean-field-type game?

First, the setup considered here is a strategic game because the producers are coupled through the price functional. Second, two conditional mean-field terms are involved:

- the conditional price $\bar{p}(t)$ based on the observations of the common noise B_o up to t.
- the square of the conditional control action

$$\bar{u}_i^2(t) = \left[\mathbb{E}(u_i \mid \mathcal{F}_t^{B_o})\right]^2$$

based on the observations of the common noise B_o up to t.

These mean-field terms make the problem a game problem of conditional mean-field type. The resulting price dynamics is of conditional mean-field type.

A strategy is a mapping that is progressively measurable to respect the information available to the producer. It maps an information set of the producer to its set of control action $\mathbb{R}_+$. Let $\mathcal{U}_i$ be the set of strategies of producer i.

Definition: [BR_i : Best Response of producer i] Any strategy $u_i^*(\cdot) \in \mathcal{U}_i$ satisfying the infimum in (2)

$$\begin{cases} \sup_{u_i \in \mathcal{U}_i} \mathbb{E}\left[R_{i,\mathcal{T}}(p_0, u)\right], \\ \mathrm{d}p = s\left[a - D - p\right]\mathrm{d}t \\ \qquad + \left(\sigma \mathrm{d}B + \int_\Theta \mu(\theta)\tilde{N}(\mathrm{d}t, \mathrm{d}\theta)\right) + \sigma_o \mathrm{d}B_o, \\ p_0, \end{cases} \tag{2}$$

is called a best-response strategy of producer i to the other producers strategy $u_{-i} \in \prod_{j\neq i}\mathcal{U}_j$. The set of best-response strategies of i is denoted by $BR_i : \prod_{j\neq i}\mathcal{U}_j \to 2^{\mathcal{U}_i}$ where $2^{\mathcal{U}_i}$ denotes the set of subsets of $\mathcal{U}_i$. ♢

B. Finding state-and-mean-field feedback Nash solution

The study of open-loop equilibrium in (deterministic) differential games goes back at least to Roos (1925 [3], pages 163-164). In the open-loop setting, the information structure of the producer is restricted to the common noise B_o and producers are allowed to employ production output functions of time and that are $\mathcal{F}^{p_0,B_o}-$measurable. This means the control action law is in the form $\phi_i(t, p_0, B_o)$. It does not explicitly depend on the price $p(t)$. For the integrability of the cost functional we impose an integrability condition on u_i^2. The set of square integrable, progressively $\mathcal{F}^{p_0,B_o}-$measurable open-loop strategies is $\mathcal{U}_i^{ol} = L^2_{\mathcal{F}^{p_0,B_o}}\left([t_0, t_1], \mathbb{R}\right)$.

In this subsection the information structure of the producer is the the price process in addition to the common noise. Therefore we look for state-and-mean-field feedback strategies. The producer is allowed to employ production output functions of time and that are $\mathcal{F}^{p_0,p,B_o}-$measurable. This means the control action law is in the form $\phi_i\left(t, p_0, p, B_o\right)$. At each time t the realized price $p(t)$ will be observed by the agent. The set of square integrable, progressively $\mathcal{F}^{p_0,p,B_o}-$measurable feedback strategies is $\mathcal{U}_i^{fb} = L^2_{\mathcal{F}^{p_0,p,B_o}}\left([t_0, t_1] \times \mathbb{R}, \mathbb{R}\right)$. A state-and-mean-field feedback Nash equilibrium is a strategy profile $u_i \in \mathcal{U}_i^{fb}$ such that $u_i \in BR_i((u_j)_{j\neq i})$ and $u_i(t)$ can be expressed as a function of $(t, p(t), \bar{p}(t))$.

Dynamics for the conditional price

Since the conditional price appears in the cost function, we would like to derive a simple equation for $\bar{p}$ as well. From the price dynamics we deduce that the conditional price solves the following stochastic differential equation:

$$\mathrm{d}\bar{p} = s\left[a - \bar{D} - \bar{p}\right]\mathrm{d}t + \sigma_o \mathrm{d}B_o,\ \bar{p}(t_0) = \bar{p}_0,$$

where $\bar{D} = \sum_i \bar{u}_i$ is the conditional expectation of the supply. Thus, the mean-field term $\bar{p}$ is a stochastic process driven by the common noise B_o. For given $\bar{D}$, this is an OU (Ornstein-Uhlenbeck) process, stabilizing around the trajectory of $a - D$ as time t_1 gets larger.

Proposition: Generically, the problem has the following interior solution (if any):
Equilibrium strategy in state-and-conditional mean-field feedback form:

$$u_i^* = -\frac{s\tilde{\alpha}_i}{r_i}(p-\bar{p}) + \frac{\bar{p}(1-s\tilde{\beta}_i)-(c_i+s\tilde{\gamma}_i)}{r_i+\bar{r}_i},$$

Conditional equilibrium price:

$$\begin{cases} \mathrm{d}\bar{p} = s\left\{a + \sum_{j=1}^{n}\frac{c_j+s\tilde{\gamma}_j}{r_j+\bar{r}_j} - \bar{p}\left(1+\sum_{j=1}^{n}\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j}\right)\right\}\mathrm{d}t \\ \qquad\qquad +\sigma_o \mathrm{d}B_o, \\ \bar{p}(t_0) = \bar{p}_0, \end{cases}$$

Stochastic Riccati system:

$$\begin{cases} \mathrm{d}\tilde{\alpha}_i = \left\{(\lambda_i+2s)\tilde{\alpha}_i - \frac{s^2}{r_i}\tilde{\alpha}_i^2 - 2s^2\tilde{\alpha}_i\sum_{j\neq i}\frac{\tilde{\alpha}_j}{r_j}\right\}\mathrm{d}t \\ \qquad\qquad +\tilde{\alpha}_{i,o}\mathrm{d}B_o, \\ \tilde{\alpha}_i(t_1) = -q, \\ \mathrm{d}\tilde{\beta}_i = \left\{(\lambda_i+2s)\tilde{\beta}_i - \frac{(1-s\tilde{\beta}_i)^2}{r_i+\bar{r}_i} + 2s\tilde{\beta}_i\sum_{j\neq i}\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j}\right\}\mathrm{d}t \\ \qquad\qquad +\tilde{\beta}_{i,o}\mathrm{d}B_o, \\ \tilde{\beta}_i(t_1) = 0, \\ \mathrm{d}\tilde{\gamma}_i = \left\{(\lambda_i+s)\tilde{\gamma}_i - s\tilde{\beta}_i a - \tilde{\beta}_{i,o}\sigma_o \right. \\ \qquad + \frac{(1-s\tilde{\beta}_i)(c_i+s\tilde{\gamma}_i)}{r_i+\bar{r}_i} + s\tilde{\gamma}_i\sum_{j\neq i}\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j} \\ \qquad \left. - s\tilde{\beta}_i\sum_{j\neq i}\frac{c_j+s\tilde{\gamma}_j}{r_j+\bar{r}_j}\right\}\mathrm{d}t - \tilde{\beta}_i\sigma_o\mathrm{d}B_o, \\ \tilde{\gamma}_i(0) = 0, \\ \mathrm{d}\tilde{\delta}_i = -\left\{-\lambda_i\tilde{\delta}_i + \frac{1}{2}\sigma_o^2\tilde{\beta}_i + \frac{1}{2}\tilde{\alpha}_i\left(\sigma^2 + \int_\Theta \mu^2(\theta)\nu(d\theta)\right)\right. \\ \qquad + s\tilde{\gamma}_i a + \tilde{\gamma}_{i,o}\sigma_o + \frac{1}{2}\frac{(c_i+s\tilde{\gamma}_i)^2}{r_i+\bar{r}_i} \\ \qquad \left. + s\tilde{\gamma}_i\sum_{j\neq i}\frac{c_j+s\tilde{\gamma}_j}{r_j+\bar{r}_j}\right\}\mathrm{d}t - \sigma_o\tilde{\gamma}_i\mathrm{d}B_o, \\ \tilde{\delta}_i(t_1) = 0, \end{cases}$$

Equilibrium revenue of producer i:

$$\mathbb{E}\frac{1}{2}\alpha_i(t_0)(p(t_0)-\bar{p}_0)^2 + \frac{1}{2}\beta_i(t_0)\bar{p}_0^2 + \gamma_i(t_0)\bar{p}_0 + \delta_i(t_0).$$

Proof: In Appendix. ∎

Taking the expected value of the conditional equilibrium revenue, we obtain

$$\frac{1}{2}\alpha_i(t_0)\mathrm{var}(p(t_0)) + \frac{1}{2}\mathbb{E}\beta_i(t_0)\bar{p}_0^2 + \mathbb{E}\gamma_i(t_0)\bar{p}_0 + \mathbb{E}\delta_i(t_0)$$

III. Larger Horizon and Stationary Solution

We consider in this section the maximization of the performance criterion $\lim_{t_1-t_0\to+\infty}\mathbb{E}R_{\mathcal{T}}(p_0,u)$. We are interested in studying Nash equilibria within the class of linear time-invariant strategies.

Limiting feedback strategies

Denoting a set of feedback Nash equilibrium strategies for the finite horizon game $u_{i,\mathcal{T}}^{fb}$, and those for the infinite horizon horizon game by $u_{i,+\infty}^{fb}$. We will study the relationship between $u_{i,\mathcal{T}}^{fb}$ and $u_{i,+\infty}^{fb}$.

As $t_1 \to +\infty$, the coefficient α_i vanishes and the (unconstrained) equilibrium strategies becomes

$$\begin{cases} u_{i,+\infty}^{fb} = \frac{\bar{p}(1-s\tilde{\beta}_i)-(c_i+s\tilde{\gamma}_i)}{r_i+\bar{r}_i}, \\ (-\lambda_i-2s)\tilde{\beta}_i + \frac{(1-s\tilde{\beta}_i)^2}{r_i+\bar{r}_i} - 2s\tilde{\beta}_i\sum_{j\neq i}\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j} = 0, \\ \tilde{\beta}_{i,o} = 0, \end{cases}$$

Producer i will participate if the price is interesting enough to get a some revenue. Let $\underline{p}_i := \frac{c_i+s\tilde{\gamma}_i}{1-s\tilde{\beta}_i}$. If in addition, σ_o vanishes then the following structure

$$u_i^* = \begin{cases} \frac{\bar{p}(1-s\tilde{\beta}_i^*)-(c_i+s\tilde{\gamma}_i^*)}{r_i+\bar{r}_i} & \text{if } \bar{p} \geq \underline{p}_i, \\ 0 & \text{otherwise} \end{cases}$$

provides a stationary equilibrium. When the price increases the equilibrium output of the producer i increases because $1 - s\tilde{\beta}_i^* > 0$. There is a stationary equilibrium price and it is given by

$$\bar{p}^* = \left(a + \sum_{j=1}^{n}\frac{c_j+s\tilde{\gamma}_j}{r_j+\bar{r}_j}\right) \Big/ \left(1+\sum_{j=1}^{n}\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j}\right). \quad (3)$$

Moreover, if the game starts at $\bar{p}_0 \neq \bar{p}^*$ the closed-loop equilibrium price converges exponentially to stationary equilibrium price. The learning dynamics stabilizes quickly to the steady state price. Note that when the speed s goes to infinity $s\beta_i$ has a limit β_i^* and the limiting electricity price is

$$p(s\to\infty) = \left(a + \sum_{j=1}^{n}\frac{c_j+\tilde{\gamma}_j^*}{r_j+\bar{r}_j}\right) \Big/ \left(1+\sum_{j=1}^{n}\frac{1-\tilde{\beta}_j^*}{r_j+\bar{r}_j}\right). \quad (4)$$

Remark: Notice that the result presented in this paper can be extended to linear coefficients for σ, $\mu(\theta)$, and σ_0. ♢

IV. Static Cournot Competition Equilibrium Price

We are interested in investigating the implications of considering the dynamic learning price model as in (1) with respect to the static price scenario, i.e., considering $p = a - D$. We compute the static revenue of producer i being

$$R_{i,\mathcal{T}}(u) = \{[a-D]u_i - C_i(u_i)\}\Lambda_i,$$

where $\Lambda_i = \int_{t_0}^{t_1} e^{-\lambda_i t} dt$,

$$C_i(u_i) = c_i u_i + \frac{1}{2}(r_i + \bar{r}_i) u_i^2.$$

Hence, the interior best response (if any) of the static Cournot game is given by

$$u_i^* \in \arg\max_{u_i \in \mathbb{R}} \{(a - D)u_i - C_i(u_i)\}, \\ = \frac{a - \sum_{j \neq i} u_j - c_i}{2 + r_i + \bar{r}_i}.$$

Hence, the static Cournot equilibrium price is given by

$$p^{\mathrm{C}} = \left(a + \sum_{j=1}^{n} \frac{c_j}{1 + r_j + \bar{r}_j}\right) / \left(1 + \sum_{j=1}^{n} \frac{1}{1 + r_j + \bar{r}_j}\right). \quad (5)$$

Finally, notice that the presented analysis allows us to determine if the consideration of the price dynamics (1) in the market is beneficial for the consumers. This is evaluated by comparing the price in (4) and (5).

V. Decision Support

In practice it is often not desirable to change continuously the price. Thus, the above "virtual" price adjustment model will be used as a learning feature to help the decision-maker before acting on the "real" price for the next time-block. For example, if the price is decided every three months cycle then this base model can be used to determine the electricity market trends and tendencies. If the price is almost in real-time, this simple adjustment technique can also be used with higher speed s.

VI. Concluding remarks

In this paper we have examined a price formation in smart energy systems using a price dynamics model introduced by Roos in 1925. We have introduced a common noise in the environment. Since the common noise is observed, producers can condition on it and exploit that information. The new conditioned price affects the revenues of the producers. We have provided explicit solution to the master system of corresponding mean-field-type game with jump-diffusion and common noise. It is shown that the optimal strategies are in state-and-mean-field feedback form.

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Appendix

Proof Proposition: We provide a proof of the main result of the paper. To do so, we first write a generic structure of the discounted revenue functional, with unknown deterministic functions to be identified. Inspired from the structure of the terminal revenue function, we try a guess functional in a quadratic form. Let

$$f_i(t,p) = \frac{1}{2}\alpha_i(t)(p - \bar{p})^2 + \frac{1}{2}\beta_i(t)\bar{p}^2 + \gamma_i(t)\bar{p} + \delta_i(t),$$

where $\alpha,\ \beta, \gamma, \delta$ are random functions of time t, that are $\mathcal{F}_t^{B_o}-$measurable such that

$$f_i(t_1, p(t_1)) = -\frac{q}{2} e^{-\lambda_i t_1}(p(t_1) - \bar{p}(t_1))^2,$$

i.e., $\alpha_i(t_1) = -q e^{-\lambda_i t_1},\ \beta_i(t_1) = \gamma_i(t_1) = \delta_i(t_1) = 0$. Using Itô's formula for jump-diffusion process one obtains

$$\mathrm{d}\bar{p}^2 = \left(2s\bar{p}(a - \bar{D} - \bar{p}) + \sigma_o^2 s\right)\mathrm{d}t + 2\bar{p}\sigma_o \mathrm{d}B_o, \quad (6)$$

$$\mathrm{d}\left[\frac{\beta_i \bar{p}^2}{2}\right] = \frac{\bar{p}^2 \mathrm{d}\beta_i}{2} + \frac{\beta_i \mathrm{d}[\bar{p}^2]}{2} + \beta_{i,o}\bar{p}\sigma_o \mathrm{d}t, \quad (7)$$

therefore, replacing (6) in (7) yields

$$\mathrm{d}\left[\frac{\beta_i \bar{p}^2}{2}\right] = \frac{1}{2}\bar{p}^2 \mathrm{d}\beta_i + \frac{1}{2}\beta_i\left(2s\bar{p}\left(a - \bar{D} - \bar{p}\right) + \sigma_o^2\right)\mathrm{d}t \\ + \beta_i \bar{p}\sigma_o \mathrm{d}B_o + \beta_{i,o}\bar{p}\sigma_o \mathrm{d}t.$$

We compute the difference between p and $\bar{p}$.

$$\mathrm{d}\left[p - \bar{p}\right] = -s\left(D - \bar{D} + p - \bar{p}\right)\mathrm{d}t \\ + \left(\sigma \mathrm{d}B + \int_\Theta \mu(\theta)\tilde{N}(\mathrm{d}t, \mathrm{d}\theta)\right),$$

$$\mathrm{d}\left[p - \bar{p}\right]^2 = 2(p - \bar{p})\left(\sigma \mathrm{d}B + \int_\Theta \mu(\theta)\tilde{N}(\mathrm{d}t, \mathrm{d}\theta)\right) + \\ -2s(p - \bar{p})(D - \bar{D} + p - \bar{p})\mathrm{d}t + \left(\sigma^2 + \int_\Theta \mu^2(\theta)\nu(\mathrm{d}\theta)\right)\mathrm{d}t. \quad (8)$$

Moreover,

$$\mathrm{d}\left[\frac{\alpha_i(p - \bar{p})^2}{2}\right] = \frac{(p - \bar{p})^2}{2}\mathrm{d}\alpha_i + \frac{1}{2}\alpha_i \mathrm{d}\left[(p - \bar{p})^2\right] + 0, \quad (9)$$

and replacing (8) in (9) yields

$$\mathrm{d}\left[\frac{\alpha_i(p - \bar{p})^2}{2}\right] = \frac{(p - \bar{p})^2}{2}\mathrm{d}\alpha_i + \frac{1}{2}\alpha_i\left(\sigma^2 + \int_\Theta \mu^2(\theta)\nu(\mathrm{d}\theta)\right)\mathrm{d}t \\ - s(p - \bar{p})\alpha_i(D - \bar{D} + p - \bar{p})\mathrm{d}t + (p - \bar{p})\alpha_i \sigma \mathrm{d}B \\ + (p - \bar{p})\alpha_i \int_\Theta \mu(\theta)\tilde{N}(\mathrm{d}t, \mathrm{d}\theta).$$

Finally,

$$\mathrm{d}[\gamma_i\bar{p}] = \bar{p}\mathrm{d}\gamma_i + \left(s\gamma_i\left(a-\bar{D}-\bar{p}\right)+\gamma_{i,o}\sigma_o\right)\mathrm{d}t + \sigma_o\gamma_i \mathrm{d}B_o.$$

Taking the conditional expectation with respect to the filtration $\mathcal{F}_{t_1}^{B_o}$ we arrive at

$$\begin{aligned}
&\mathbb{E}\left[f_i(t_1,p(t_1)) - f_i(t_0,p(t_0))|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&= \frac{1}{2}\int_{t_0}^{t_1}(p-\bar{p})^2\left(\mathrm{d}\alpha_i - 2s\alpha_i \mathrm{d}t\right) - 2s\alpha_i(p-\bar{p})(D-\bar{D})\mathrm{d}t\\
&+\frac{1}{2}\int_{t_0}^{t_1}\alpha_i\left(\sigma^2+\int\mu^2\nu(\mathrm{d}\theta)\right)\mathrm{d}t + \frac{1}{2}\int_{t_0}^{t_1}\bar{p}^2\left(\mathrm{d}\beta_i - 2s\beta_i\mathrm{d}t\right)\\
&+\frac{1}{2}\int_{t_0}^{t_1} 2s\beta_i\bar{p}(a-\bar{D})\mathrm{d}t + (\sigma_o^2\beta_i + 2\beta_{i,o}\bar{p}\sigma_o)\mathrm{d}t\\
&+\frac{1}{2}\int_{t_0}^{t_1} 2\beta_i\bar{p}\sigma_o \mathrm{d}B_o + \int_{t_0}^{t_1}\bar{p}\left(\mathrm{d}\gamma_i - s\gamma_i \mathrm{d}t\right)\\
&+\int_{t_0}^{t_1}\left(s\gamma_i(a-\bar{D})+\gamma_{i,o}\sigma_o\right)\mathrm{d}t + \int_{t_0}^{t_1}\sigma_o\gamma_i \mathrm{d}B_o + \mathrm{d}\delta_i,
\end{aligned}$$

The expected revenue can be expressed as

$$\begin{aligned}
&e^{-\lambda_i t}\mathbb{E}\left[\bar{p}u_i - C_i(u_i)|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&\quad= e^{-\lambda_i t}\mathbb{E}\left[(\bar{p}-c_i)\bar{u}_i|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&\quad -e^{-\lambda_i t}\mathbb{E}\left[\tfrac{1}{2}r_i(u_i-\bar{u}_i)^2 + \tfrac{1}{2}(r_i+\bar{r}_i)(\bar{u}_i)^2|\ \mathcal{F}_{t_1}^{B_o}\right],
\end{aligned}$$

We now express the difference between the long-term revenue and the guess functional.

$$\begin{aligned}
&\mathbb{E}[R_{i,\mathcal{T}} - f_i(t_0,p(t_0))|\ \mathcal{F}_{t_1}^{B_o}]\\
&= -\frac{q}{2}e^{-\lambda_i t_1}\mathbb{E}\left[(p(t_1)-\bar{p}(t_1))^2|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&+\frac{1}{2}\mathbb{E}\int_{t_0}^{t_1} e^{-\lambda_i t}\left(2(\bar{p}-c_i)\bar{u}_i - r_i(u_i-\bar{u}_i)^2\right.\\
&\left.-(r_i+\bar{r}_i)(\bar{u}_i)^2|\ \mathcal{F}_{t_1}^{B_o}\right)\mathrm{d}t + \mathbb{E}\frac{1}{2}\int_{t_0}^{t_1}(p-\bar{p})^2(\mathrm{d}\alpha_i - 2s\alpha_i\mathrm{d}t)\\
&+\frac{1}{2}\int_{t_0}^{t_1} -2s\alpha_i(p-\bar{p})(D-\bar{D})\mathrm{d}t + \alpha_i\left(\sigma^2+\int_\Theta \mu^2(\theta)\nu(\mathrm{d}\theta)\right)\mathrm{d}t\\
&+\frac{1}{2}\int_{t_0}^{t_1}\bar{p}^2\left(\mathrm{d}\beta_i - 2s\beta_i\mathrm{d}t\right) + 2s\beta_i\bar{p}(a-\bar{D})\mathrm{d}t\\
&+\frac{1}{2}\int_{t_0}^{t_1}\left(\sigma_o^2\beta_i + 2\beta_{i,o}\bar{p}\sigma_o\right)\mathrm{d}t + 2\beta_i\bar{p}\sigma_o\mathrm{d}B_o\\
&+\int_{t_0}^{t_1}\bar{p}\left(\mathrm{d}\gamma_i - s\gamma_i\mathrm{d}t\right) + [s\gamma_i(a-\bar{D})+\gamma_{i,o}\sigma_o]\mathrm{d}t\\
&+\int_{t_0}^{t_1}\sigma_o\gamma_i\mathrm{d}B_o + \mathrm{d}\delta_i.
\end{aligned}$$

Therefore

$$\begin{aligned}
&\mathbb{E}\left[R_{i,\mathcal{T}} - f_i(t_0,p(t_0))|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&= \frac{(\alpha_i(t_1) - qe^{-\lambda_i t_1})}{2}\mathbb{E}\left[(p(t_1)-\bar{p}(t_1))^2|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&+\mathbb{E}\int_{t_0}^{t_1}\frac{e^{-\lambda_i t}}{2}\left(2(\bar{p}-c_i)\bar{u}_i - r_i(u_i-\bar{u}_i)^2 - (r_i+\bar{r}_i)(\bar{u}_i)^2\right)\\
&-2s\alpha_i(p-\bar{p})(D-\bar{D}) - 2s(\beta_i\bar{p}+\gamma_i)\bar{D}\mathrm{d}t\\
&+\mathbb{E}\int_{t_0}^{t_1}\frac{(p-\bar{p})^2}{2}(\mathrm{d}\alpha_i - 2s\alpha_i\mathrm{d}t) + \frac{\alpha_i}{2}\left(\sigma^2+\int_\Theta\mu^2(\theta)\nu(\mathrm{d}\theta)\right)\mathrm{d}t\\
&+\frac{1}{2}\int_{t_0}^{t_1}\bar{p}^2\left(\mathrm{d}\beta_i - 2s\beta_i\mathrm{d}t\right) + 2s\beta_i\bar{p}a\,\mathrm{d}t\\
&+\frac{1}{2}\int_{t_0}^{t_1}(\sigma_o^2\beta_i + 2\beta_{i,o}\bar{p}\sigma_o)\mathrm{d}t + 2\beta_i\bar{p}\sigma_o\mathrm{d}B_o\\
&+\int_{t_0}^{t_1}\bar{p}\left(\mathrm{d}\gamma_i - s\gamma_i\mathrm{d}t\right) + [s\gamma_i a + \gamma_{i,o}\sigma_o]\mathrm{d}t\\
&+\int_{t_0}^{t_1}\sigma_o\gamma_i\mathrm{d}B_o + \mathrm{d}\delta_i,
\end{aligned}$$

We now use a completion of square technique. Therefore, the term

$$\begin{aligned}
&e^{-\lambda_i t}\left(2(\bar{p}-c_i)\bar{u}_i - r_i(u_i-\bar{u}_i)^2 - (r_i+\bar{r}_i)(\bar{u}_i)^2\right)\\
&\qquad - 2s\alpha_i(p-\bar{p})(D-\bar{D}) - 2s(\beta_i\bar{p}+\gamma_i)\bar{D}
\end{aligned}$$

is equal to

$$\begin{aligned}
&-(r_i+\bar{r}_i)\,e^{-\lambda_i t}(\bar{u}_i)^2 + 2\left[(\bar{p}-c_i)e^{-\lambda_i t} - s(\beta_i\bar{p}+\gamma_i)\right]\bar{u}_i\\
&\quad - r_i e^{-\lambda_i t}(u_i-\bar{u}_i)^2 - 2s\alpha_i(p-\bar{p})(u_i-\bar{u}_i)\\
&\qquad - 2s\alpha_i(p-\bar{p})(D_{-i}-\bar{D}_{-i}) - 2s(\beta_i\bar{p}+\gamma_i)\bar{D}_{-i}\\
&= -(r_i+\bar{r}_i)e^{-\lambda_i t}\left(u_i - \frac{\bar{p}(e^{-\lambda_i t}-s\beta_i)-(c_ie^{-\lambda_i t}+s\gamma_i)}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\right)^2\\
&- r_ie^{-\lambda_i t}\left(u_i - \bar{u}_i + \frac{s\alpha_i(p-\bar{p})}{r_ie^{-\lambda_i t}}\right)^2 + (p-\bar{p})^2\left(\frac{s^2\alpha_i^2}{r_ie^{-\lambda_i t}}\right.\\
&\left.+2s^2\alpha_i\sum_{j\neq i}\frac{\alpha_j}{r_je^{-\lambda_j t}}\right) + \bar{p}^2\left(\frac{(e^{-\lambda_i t}-s\beta_i)^2}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\right.\\
&\left.-2s\beta_i\sum_{j\neq i}\frac{e^{-\lambda_j t}-s\beta_j}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\right) - 2\bar{p}\left(\frac{(e^{-\lambda_i t}-s\beta_i)(c_ie^{-\lambda_i t}+s\gamma_i)}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\right.\\
&\qquad\left.+s\gamma_i\sum_{j\neq i}\frac{(e^{-\lambda_j t}-s\beta_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}} - s\beta_i\sum_{j\neq i}\frac{(c_je^{-\lambda_j t}+s\gamma_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\right)\\
&+\frac{(c_ie^{-\lambda_i t}+s\gamma_i)^2}{(r_i+\bar{r}_i)e^{-\lambda_i t}} + 2s\gamma_i\sum_{j\neq i}\frac{(c_je^{-\lambda_j t}+s\gamma_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}.
\end{aligned}$$

where we used $D_{-i} - \bar{D}_{-i} = -s(p-\bar{p})\sum_{j\neq i}\frac{\alpha_j}{r_je^{-\lambda_j t}}$, and

$$\bar{D}_{-i} = \bar{p}\sum_{j\neq i}\frac{e^{-\lambda_j t}-s\beta_j}{(r_j+\bar{r}_j)e^{-\lambda_j t}} - \sum_{j\neq i}\frac{c_je^{-\lambda_j t}+s\gamma_j}{(r_j+\bar{r}_j)e^{-\lambda_j t}}.$$

It follows that

$$\begin{aligned}
&\mathbb{E}\left[R_{i,\mathcal{T}} - f_i(t_0,p(t_0))|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&= \frac{\alpha_i(t_1) - qe^{-\lambda_i t_1}}{2}\mathbb{E}\left[(p(t_1)-\bar{p}(t_1))^2|\ \mathcal{F}_{t_1}^{B_o}\right]\\
&-\mathbb{E}\int_{t_0}^{t_1}\frac{r_i+\bar{r}_i}{2}e^{-\lambda_i t}\left(\bar{u}_i - \frac{\bar{p}(e^{-\lambda_i t}-s\beta_i)-(c_ie^{-\lambda_i t}+s\gamma_i)}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\right)^2\mathrm{d}t\\
&-\mathbb{E}\int_{t_0}^{t_1}\frac{r_i}{2}e^{-\lambda_i t}\left(u_i-\bar{u}_i+\frac{s\alpha_i(p-\bar{p})}{r_ie^{-\lambda_i t}}\right)^2 dt\\
&+\mathbb{E}\int_{t_0}^{t_1}\frac{(p-\bar{p})^2}{2}\left\{d\alpha_i + \left(-2s\alpha_i + \frac{s^2\alpha_i^2}{r_ie^{-\lambda_i t}}\right.\right.\\
&\qquad\qquad\left.\left.+2s^2\alpha_i\sum_{j\neq i}\frac{\alpha_j}{r_je^{-\lambda_j t}}\right)\mathrm{d}t\right\}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{2}\int_{t_0}^{t_1}\bar{p}^2\Big\{d\beta_i+(-2s\beta_i+\frac{(e^{-\lambda_i t}-s\beta_i)^2}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\\
&\qquad -2s\beta_i\sum_{j\neq i}\frac{(e^{-\lambda_j t}-s\beta_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}})\mathrm{d}t\Big\}\\
&+\int_{t_0}^{t_1}\bar{p}\,\{\mathrm{d}\gamma_i-s\gamma_i\mathrm{d}t+s\beta_i a\mathrm{d}t+\beta_{i,o}\sigma_o\mathrm{d}t\\
&\qquad +\beta_i\sigma_o\mathrm{d}B_o-\frac{(e^{-\lambda_i t}-s\beta_i)(c_ie^{-\lambda_i t}+s\gamma_i)}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\\
&\qquad -s\gamma_i\sum_{j\neq i}\frac{(e^{-\lambda_j t}-s\beta_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}+s\beta_i\sum_{j\neq i}\frac{(c_je^{-\lambda_j t}+s\gamma_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\Big\}\\
&+\int_{t_0}^{t_1}\mathrm{d}\delta_i+\Big[\frac{1}{2}\sigma_o^2\beta_i+\frac{1}{2}\alpha_i\left(\sigma^2+\int_\Theta\mu^2(\theta)\nu(\mathrm{d}\theta)\right)\\
&\qquad +s\gamma_i a+\gamma_{i,o}\sigma_o\Big]\mathrm{d}t+\sigma_o\gamma_i\mathrm{d}B_o\\
&\qquad +\frac{1}{2}\frac{(c_ie^{-\lambda_i t}+s\gamma_i)^2}{(r_i+\bar{r}_i)e^{-\lambda_i t}}+s\gamma_i\sum_{j\neq i}\frac{(c_je^{-\lambda_j t}+s\gamma_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}.
\end{aligned}
$$

We deduce that $\mathbb{E}\left[R_{i,\mathcal{T}}-f_i(t_0,p(t_0))|\ \mathcal{F}_{t_1}^{B_o}\right]\leq 0$ and the equality occurs when the random coefficient solves the following system:

$$\bar{u}_i^*=\frac{\bar{p}(e^{-\lambda_i t}-s\beta_i)-(c_ie^{-\lambda_i t}+s\gamma_i)}{(r_i+\bar{r}_i)e^{-\lambda_i t}},$$

Equilibrium strategy in state-and-mean-field feedback form:

$$
\begin{aligned}
u_i^*&=-\frac{s\alpha_i(p-\bar{p})}{r_ie^{-\lambda_i t}}+\bar{u}_i^*,\\
&=-\frac{s\alpha_i}{r_ie^{-\lambda_i t}}(p-\bar{p})+\frac{\bar{p}(e^{-\lambda_i t}-s\beta_i)-(c_ie^{-\lambda_i t}+s\gamma_i)}{(r_i+\bar{r}_i)e^{-\lambda_i t}},
\end{aligned}
$$

Conditional equilibrium price:

$$
\begin{cases}
\mathrm{d}\bar{p}=s\Bigg[a+\sum_{j=1}^n\frac{c_je^{-\lambda_j t}+s\gamma_j}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\\
\qquad -\bar{p}\left(1+\sum_{j=1}^n\frac{e^{-\lambda_j t}-s\beta_j}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\right)\Bigg]\mathrm{d}t+\sigma_o\mathrm{d}B_o,\\
\bar{p}(t_0)=\bar{p}_0,
\end{cases}
$$

Stochastic Riccati system:

$$
\begin{cases}
\mathrm{d}\alpha_i=-\left(-2s\alpha_i+\frac{s^2\alpha_i^2}{r_ie^{-\lambda_i t}}+2s^2\alpha_i\sum_{j\neq i}\frac{\alpha_j}{r_je^{-\lambda_j t}}\right)\mathrm{d}t\\
\qquad +\alpha_{i,o}\mathrm{d}B_o,\\
\alpha_i(t_1)=-qe^{-\lambda_i t_1},\\
\mathrm{d}\beta_i=-\Bigg(-2s\beta_i+\frac{(e^{-\lambda_i t}-s\beta_i)^2}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\\
\qquad -2s\beta_i\sum_{j\neq i}\frac{(e^{-\lambda_j t}-s\beta_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\Bigg)\mathrm{d}t+\beta_{i,o}\mathrm{d}B_o,\\
\beta_i(t_1)=0,\\
\mathrm{d}\gamma_i=\Big\{s\gamma_i-s\beta_i a-\beta_{i,o}\sigma_o\\
\qquad +\frac{(e^{-\lambda_i t}-s\beta_i)(c_ie^{-\lambda_i t}+s\gamma_i)}{(r_i+\bar{r}_i)e^{-\lambda_i t}}+s\gamma_i\sum_{j\neq i}\frac{(e^{-\lambda_j t}-s\beta_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\\
\qquad -s\beta_i\sum_{j\neq i}\frac{(c_je^{-\lambda_j t}+s\gamma_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\Big\}\mathrm{d}t-\beta_i\sigma_o\mathrm{d}B_o,\\
\gamma_i(0)=0,\\
\mathrm{d}\delta_i=-\Big\{\frac{1}{2}\sigma_o^2\beta_i+\frac{1}{2}\alpha_i(\sigma^2+\int_\Theta\mu^2(\theta)\nu(\mathrm{d}\theta))\\
\qquad +s\gamma_i a+\gamma_{i,o}\sigma_o+\frac{1}{2}\frac{(c_ie^{-\lambda_i t}+s\gamma_i)^2}{(r_i+\bar{r}_i)e^{-\lambda_i t}}\\
\qquad +s\gamma_i\sum_{j\neq i}\frac{(c_je^{-\lambda_j t}+s\gamma_j)}{(r_j+\bar{r}_j)e^{-\lambda_j t}}\Big\}\mathrm{d}t-\sigma_o\gamma_i\mathrm{d}B_o,\\
\delta_i(t_1)=0,
\end{cases}
$$

Conditional equilibrium revenue: $\frac{1}{2}\alpha_i(t_0)(p(t_0)-\bar{p}_0)^2+\frac{1}{2}\beta_i(t_0)\bar{p}_0^2+\gamma_i(t_0)\bar{p}_0+\delta_i(t_0)$. Set $\tilde{\alpha}_i=\alpha_ie^{\lambda_i t},\ \tilde{\beta}_i=\beta_ie^{\lambda_i t}$. Then, Equilibrium strategy in state-and-mean-field feedback form:

$$u_i^*=-\frac{s\tilde{\alpha}_i}{r_i}(p-\bar{p})+\frac{\bar{p}(1-s\tilde{\beta}_i)-(c_i+s\tilde{\gamma}_i)}{r_i+\bar{r}_i},$$

Conditional equilibrium price:

$$
\begin{cases}
\mathrm{d}\bar{p}=s\left\{a+\sum_{j=1}^n\frac{c_j+s\tilde{\gamma}_j}{r_j+\bar{r}_j}-\bar{p}\left(1+\sum_{j=1}^n\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j}\right)\right\}\mathrm{d}t\\
\qquad +\sigma_o\mathrm{d}B_o,\\
\bar{p}(t_0)=\bar{p}_0,
\end{cases}
$$

Stochastic Riccati system:

$$
\begin{cases}
\mathrm{d}\tilde{\alpha}_i=\left\{(\lambda_i+2s)\tilde{\alpha}_i-\frac{s^2}{r_i}\tilde{\alpha}_i^2-2s^2\tilde{\alpha}_i\sum_{j\neq i}\frac{\tilde{\alpha}_j}{r_j}\right\}\mathrm{d}t\\
\qquad +\tilde{\alpha}_{i,o}\mathrm{d}B_o,\\
\tilde{\alpha}_i(t_1)=-q,\\
\mathrm{d}\tilde{\beta}_i=\left\{(\lambda_i+2s)\tilde{\beta}_i-\frac{(1-s\tilde{\beta}_i)^2}{r_i+\bar{r}_i}+2s\tilde{\beta}_i\sum_{j\neq i}\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j}\right\}\mathrm{d}t\\
\qquad +\tilde{\beta}_{i,o}\mathrm{d}B_o,\\
\tilde{\beta}_i(t_1)=0,\\
\mathrm{d}\tilde{\gamma}_i=\Big\{(\lambda_i+s)\tilde{\gamma}_i-s\tilde{\beta}_ia-\tilde{\beta}_{i,o}\sigma_o\\
\qquad +\frac{(1-s\tilde{\beta}_i)(c_i+s\tilde{\gamma}_i)}{r_i+\bar{r}_i}+s\tilde{\gamma}_i\sum_{j\neq i}\frac{1-s\tilde{\beta}_j}{r_j+\bar{r}_j}\\
\qquad -s\tilde{\beta}_i\sum_{j\neq i}\frac{c_j+s\tilde{\gamma}_j}{r_j+\bar{r}_j}\Big\}\mathrm{d}t-\tilde{\beta}_i\sigma_o\mathrm{d}B_o,\\
\tilde{\gamma}_i(0)=0,\\
\mathrm{d}\tilde{\delta}_i=-\Big\{-\lambda_i\tilde{\delta}_i+\frac{1}{2}\sigma_o^2\tilde{\beta}_i+\frac{1}{2}\tilde{\alpha}_i\left(\sigma^2+\int_\Theta\mu^2(\theta)\nu(\mathrm{d}\theta)\right)\\
\qquad +s\tilde{\gamma}_ia+\tilde{\gamma}_{i,o}\sigma_o+\frac{1}{2}\frac{(c_i+s\tilde{\gamma}_i)^2}{(r_i+\bar{r}_i)}\\
\qquad +s\tilde{\gamma}_i\sum_{j\neq i}\frac{(c_j+s\tilde{\gamma}_j)}{(r_j+\bar{r}_j)}\Big\}\mathrm{d}t-\sigma_o\tilde{\gamma}_i\mathrm{d}B_o,\\
\tilde{\delta}_i(t_1)=0,
\end{cases}
$$

completing the proof ■