23rd International Symposium on Mathematical Theory of Networks and Systems
Hong Kong University of Science and Technology, Hong Kong, July 16-20, 2018

Containment of second-order multi-agent systems with disturbances under independent position and velocity topologies* (Extended Abstract)

Xuxi Zhang, Xianping Liu

Abstract— **This paper considers containment control problem for second-order multi-agent systems with disturbances generated from linear exosystems and nonlinear exosystems, respectively, under independent position and velocity topologies. Firstly, for the case that the disturbances are generated from linear exosystems, linear disturbance observer and control protocol are proposed for each follower using the relative states among neighboring agents; Secondly, for the case that the disturbances are generated from nonlinear exosystems, based on dynamic-gain technique, nonlinear disturbance observer and control protocol are provided, under which all the followers are driven to the convex hull spanned by the leaders.**

I. INTRODUCTION

In the area of cooperative control of multi-agent systems, consensus plays an important role in achieving collective behavior by using only local interactions of agents. In accordance with whether or not there exist leader agents, consensus of multi-agent systems can be categorized into three classes, i.e., leaderless problems [1], leader-following problems with one leader [2] and containment problems with multiple leaders [3].

Note that the containment control problems of multi-agent systems have been explored for first-order and second-order multi-agent systems with multiple dynamical leaders by many researchers in the literature [4], [5]. In the aforementioned works, the position and velocity interactions among the agents are described by the identical topologies, which might be restrictive in some circumstances [6]. However, in many cases, the position and the velocity of the agents are usually measured by different sensors, which will lead to that the position and velocity information are communicated among agents over different network topologies. Furthermore, there is no results on containment control problem of second-order multi-agent systems with disturbances generated from some exosystems under independent position and velocity topologies, which constitutes the main motivation of the present research.

The purpose of this paper is to study the containment control problem of second-order multi-agent systems with disturbances generated from some linear exosystems/nonlinear exosystems, under independent position and velocity topologies.

*This work was supported by the National Natural Science Foundation of China (No. 61503089), and the Natural Science Foundation of Heilongjiang Province (No. QC2015077), Postdoctoral Research Startup Foundation of Heilongjiang (No. LBH-Q16068).

Xuxi Zhang and Xianping Liu are with College of Science, Harbin Engineering University, Harbin, China, 150001 zxx08@126.com, lxp@hrbeu.edu.cn.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Preliminaries

We consider a group of $N+M$ agents, whose interaction topologies of position and velocity can be represented by $\mathscr{G}^p=(V,E^p,\mathscr{A}^p)$ and $\mathscr{G}^v=(V,E^v,\mathscr{A}^v)$, respectively, with $V=\{v_1,v_1,\cdots,v_{N+M}\}$, $E^p,E^v\subseteq V\times V$, and $\mathscr{A}^p=(a_{ij})\in R^{(N+M)\times(N+M)}$ and $\mathscr{A}^v=(b_{ij})\in R^{(N+M)\times(N+M)}$. An edge (v_i,v_j) of $\mathscr{G}^p/\mathscr{G}^v$ from v_i to v_j representing that node v_j can obtain position/velocity information from node v_i, but not necessarily vice versa. $a_{ij}>0$ if and only if $(v_j,v_i)\in E^p$, and $a_{ij}=0$, otherwise. Similarly, $b_{ij}>0$ if and only if $(v_j,v_i)\in E^v$, and $b_{ij}=0$, otherwise. Let $\mathscr{N}_i^p=\{v_j:(v_j,v_i)\in E^p\}$ and $\mathscr{N}_i^v=\{v_j:(v_j,v_i)\in E^v\}$ denote the position neighbor set and the velocity neighbor set of every agent v_i, respectively. The graph $\mathscr{G}^p/\mathscr{G}^v$ is said to have a united spanning tree, if for each of the followers, there exists at least one leader having a directed path in graph $\mathscr{G}^p/\mathscr{G}^v$to the follower.

We assume that the agents 1 to N are followers, and the agents $N+1$ to $N+M$ are leaders, and that each follower has at least one neighbor, but each leader has no neighbors. Moreover, the position and velocity network topologies for the N follower agents are represented by G_f^p and G_f^v, respectively.

Let the Laplacian matrix $L_p=(l_{ij})\in R^{(N+M)\times(N+M)}$ associated with $\mathscr{G}^p$ be defined as $l_{pii}=\sum_{j=1,j\neq i}^{N+M}a_{ij},\ l_{pij}=-a_{ij},\ \ i\neq j$. Similarly, the Laplacian matrix $L_v=(l_{ij})\in R^{(N+M)\times(N+M)}$ associated with $\mathscr{G}^v$ can be depicted by $l_{vii}=\sum_{j=1,j\neq i}^{N+M}b_{ij},\ l_{vij}=-b_{ij},\ \ i\neq j$. Then, the Laplacian matrix L_p and L_v can be partitioned as

$$L_p=\begin{bmatrix}L_{p1} & L_{p2}\\ 0 & 0\end{bmatrix},L_v=\begin{bmatrix}L_{v1} & L_{v2}\\ 0 & 0\end{bmatrix}.$$

B. Problem Formulation

In this paper, we suppose that the multi-agent system consists of N followers and M leaders. To simplify notation, we use $\mathbb{F}=1,2,...,N$ and $\mathbb{L}=N+1,N+2,...,N+M$, respectively, to denote the follower set and the leader set.

Each follower of the concerned multi-agent system is represented as

$$\begin{aligned}\dot{x}_i&=v_i,\\ \dot{v}_i&=u_i+d_i,\ \ i\in\mathbb{F}\end{aligned}\tag{1}$$

where $x_i \in R^m, v_i \in R^m, u_i \in R^m$ are the position, velocity and the control input of the ith follower, respectively. And, $d_i \in R^m$ is the external disturbance.

The M leaders of the concerned multi-agent system are described by

$$\dot{x}_i = v_d, \ \ i \in \mathbb{L}, \tag{2}$$

where $x_i \in R^m, v_d \in R^m$ are the position and velocity of the leader i, respectively.

Definition 1: For the multi-agent system consists of (1) and (2), the containment control problem is solved under a certain distributed controller, if the position and velocity states of the followers asymptotically converge to the convex hull formed by those of the leaders.

Lemma 1: Assume that $\mathscr{G}^p/\mathscr{G}^v$ has a united spanning tree. Then, the following results hold true.

1) The real parts of the eigenvalues of L_{p1}/L_{v1} are all positive.
2) $-L_{p1}^{-1}L_{p2}\mathbf{1}_M = \mathbf{1}_N, \ -L_{v1}^{-1}L_{v2}\mathbf{1}_M = \mathbf{1}_N$.
3) $L_p + L_p^T > 0/L_v + L_v^T > 0$, if G_f^p/G_f^v is balanced. And, $L_{p1} > 0/L_{v1} > 0$, if G_f^p/G_f^v is undirected.

III. Containment control with disturbances generated from linear exosystems

Assume that the disturbance d_i are generated by some the following linear exosystems

$$\dot{\xi}_i = A_i\xi_i, d_i = C_i\xi_i, \ \ i \in \mathbb{F}, \tag{3}$$

where $\xi_i \in R^{m_i}$, $A_i \in R^{m_i\times m_i}, C_i \in R^{m\times m_i}$.

Assumption 1: The matrix pair (A_i, C_i) of the external disturbance system is observable

Now, a disturbance observer is developed as,

$$\begin{aligned}\dot{z}_i &= (A_i - K_iC_i)(z_i + K_iv_i) - K_iu_i, i \in \mathbb{F},\\ \hat{\xi}_i &= z_i + K_iv_i, \hat{d}_i = C_i\hat{\xi}_i.\end{aligned} \tag{4}$$

where $z_i \in R^{m_i}$ is the internal state variable of the disturbance observer (4), $\hat{\xi}_i$ and $\hat{d}_i$ are the estimations for ξ_i and d_i, respectively, and $K_i \in R^{m_i\times m}$ is the gain matrix of (4).

Next, denote

$$e_i = \xi_i - \hat{\xi}_i \tag{5}$$

as the estimation error between ξ and its estimation $\hat{\xi}_i$.

Then, it follows from eqs. (3), (4) and (5), that

$$\dot{e}_i = (A_i - K_iC_i)e_i. \tag{6}$$

Based on the estimation given by the disturbance observer (4), the control protocol u_i for the ith follower is designed as

$$u_i = \sum_{j\in\mathscr{N}_i^p} a_{ij}(x_j - x_i) + \sum_{j\in\mathscr{N}_i^v} b_{ij}(v_j - v_i) - \hat{d}_i, \ \ i \in \mathbb{F}, \tag{7}$$

where $\hat{d}_i$ denotes the estimation of the disturbance d_i.

Define

$$\begin{aligned}x_f &= \begin{bmatrix} x_1^T & x_2^T & \cdots & x_N^T\end{bmatrix}^T,\\ x_l &= \begin{bmatrix} x_{N+1}^T & x_{N+2}^T & \cdots & x_{N+M}^T\end{bmatrix}^T,\\ \bar{x}_f &= x_f + (L_{p1}^{-1}L_{p2}\otimes I_m)x_l,\\ v_f &= \begin{bmatrix} v_1^T & v_2^T & \cdots & v_N^T\end{bmatrix}^T,\\ v_l &= \mathbf{1}_N \otimes v_d,\\ \bar{v}_f &= v_f - \mathbf{1}_N \otimes v_d.\end{aligned} \tag{8}$$

Theorem 1: Consider the multi-agent system consists of (1) and (2) with the disturbance generated from linear exosystem (3). Suppose that Assumption 1 holds, G^p and G^v both contain a united spanning tree, and G_f^p is undirected. Then, the control protocol proposed by (7) solves the containment control problem if the matrices $A_i - K_iC_i, i \in \mathbb{F}$ are Hurwitz and the matrix $L_v + L_v^T$ is positive definite.

Proof: Firstly, according to the Lemma 1, and the definition of $\bar{x}_f$ and $\bar{v}_f$ in eq. (8), the containment control is achieved if $\bar{x}_f$ and $\bar{v}_f$ both converge to zero as $t \to \infty$.

To this end, substituting (7) into (1) yields

$$\begin{aligned}\dot{x}_i &= v_i,\\ \dot{v}_i &= \sum_{j\in\mathscr{N}_i^p} a_{ij}(x_j - x_i) + \sum_{j\in\mathscr{N}_i^v} b_{ij}(v_j - v_i) - \hat{d}_i + d_i.\end{aligned} \tag{9}$$

In view of the definition of $d_i, \hat{d}_i$, and e_i, eq. (9) can be rewritten as

$$\begin{aligned}\dot{x}_i &= v_i,\\ \dot{v}_i &= \sum_{j\in\mathscr{N}_i^p} a_{ij}(x_j - x_i) + \sum_{j\in\mathscr{N}_i^v} b_{ij}(v_j - v_i) + C_ie_i.\end{aligned} \tag{10}$$

Furthermore, by using the notation defined in (8), we have

$$\begin{aligned}\dot{\bar{x}}_f &= \bar{v}_f,\\ \dot{\bar{v}}_f &= -(L_{p1}\otimes I_m)\bar{x}_f - (L_{v1}\otimes I_m)\bar{v}_f + Ce,\end{aligned} \tag{11}$$

where

$$\begin{aligned}e &= \begin{bmatrix} e_1^T & e_2^T & \cdots & e_N^T\end{bmatrix}^T,\\ C &= blockdiag(C_1, \cdots, C_N).\end{aligned} \tag{12}$$

From the definition of e in eq. (12) and the dynamics of e_i in eq. (6), the global error dynamics of e can be written as

$$\dot{e} = (A - KC)e, \tag{13}$$

where

$$\begin{aligned}A &= blockdiag(A_1, \cdots, A_N),\\ K &= blockdiag(K_1, \cdots, K_N).\end{aligned}$$

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}\bar{x}_f^T(L_{p1}\otimes I_m)\bar{x}_f + \frac{1}{2}\bar{v}_f^T\bar{v}_f + \varepsilon e^TPe, \tag{14}$$

where ε is a positive real number satisfying $\varepsilon \geq \frac{\|C\|^2}{\lambda_{min}(\frac{L_v+L_v^T}{2})}$, and P is a positive definite matrix satisfying

$$P(A - KC) + (A - KC)^TP = -I. \tag{15}$$

Taking the derivative of V defined in eq. (15) with respect to time t, one can obtain that

$$\dot{V}(t) \leq -\frac{\lambda_{min}(\frac{L_v+L_v^T}{2})}{2}\|\bar{v}_f\|^2 - \frac{\varepsilon}{2}\|e\|^2. \tag{16}$$

Therefore, we conclude that $V(t)$ is bounded, so are $\bar{x}_f, \bar{v}_f$, and e. Noting that $\dot{V}(t) \equiv 0$, by inequality (16), is equivalent to $\bar{v}_f \equiv 0$ and $e \equiv 0$. Associated with system (11) and (13), we obtain that $\bar{x}_f \equiv 0$. Finally, according to the LaSalle's invariance principle, the solution to system (11) and (13) converge to the largest invariant set $\{(\bar{x}_f, \bar{v}_f, e)|\dot{V}(t) = 0\}$, which implies that $\bar{x}_f \to 0$ and $\bar{v}_f \to 0$ as $t \to \infty$. ■

IV. Containment control with disturbances generated from nonlinear exosystems

Suppose that the disturbance are generated by the the following nonlinear exosystem:

$$\dot{\xi}_i = A_i\xi_i + \phi_i(\xi_i), d_i = C_i\xi_i, \tag{17}$$

where $\xi_i \in R^{m_i}$, $A_i \in R^{m_i\times m_i}, C_i \in R^{1\times m_i}$, and $\phi_i(\cdot)$ is a globally Lipschitz function, i.e., there exists a positive constant $c_{\phi_i} > 0$, such that $\|\phi_i(\xi_{i1}) - \phi_i(\xi_{i2})\| \leq c_{\phi_i}\|\xi_{i1} - \xi_{i2}\|$, for any $\xi_{i1}, \xi_{i2} \in R^{m_i}$.

We propose the following disturbance observer

$$\begin{aligned}\dot{z}_i &= (A_i - K_iC_i)(z_i + K_iv_i) - K_iu_i + \phi_i(z_i + K_iv_i) + Q_i^{-1}\zeta_ie_i,\\ \dot{\zeta}_i &= e_i^Te_i, \hat{\xi}_i = z_i + K_iv_i, \hat{d}_i = C_i\hat{\xi}_i, e_i = \xi_i - \hat{\xi}_i,\end{aligned} \tag{18}$$

where $z_i \in R^{m_i}$, K_i is the observer gain matrix, ζ_i is the so-called dynamic gain [7], $\hat{\xi}_i$ and $\hat{d}_i$ denote the estimates of ξ_i and d_i, respectively, and e_i is the estimation error between ξ and its estimation $\hat{\xi}_i$.

From eqs. (17) and (18), the estimation error e_i is given as

$$\dot{e}_i = (A_i - K_iC_i)e_i + \phi_i(\xi_i) - \phi_i(\hat{\xi}_i) - Q_i^{-1}\zeta_ie_i. \tag{19}$$

Lemma 2: The estimation error system (19) is asymptotically stable if the gain matrix K_i is chosen such that the matrix $A_i - K_iC_i$ is Hurwitz, and Q_i is the positive definite matrix solution of the following Lyapunov equation:

$$Q_i(A_i - K_iC_i) + (A_i - K_iC_i)^TQ_i = -I. \tag{20}$$

Moreover, there is a Lyapunov function candidate V_i for (19) satisfying

$$\dot{V}_i \leq -\varepsilon\|e_i\|^2, \tag{21}$$

with $\varepsilon > 1$.

Next, the control protocol u_i for the ith follower is presented as

$$u_i = \sum_{j\in\mathcal{N}_i^p} a_{ij}(x_j - x_i) + \sum_{j\in\mathcal{N}_i^v} b_{ij}(v_j - v_i) - \hat{d}_i, \ \ i \in \mathbb{F}, \tag{22}$$

where $\hat{d}_i$, which is produced by the nonlinear disturbance observer (18), denotes the estimation of the disturbance d_i.

Substituting (22) into (1), we have

$$\begin{aligned}\dot{x}_i &= v_i,\\ \dot{v}_i &= \sum_{j\in\mathcal{N}_i^p} a_{ij}(x_j - x_i) + \sum_{j\in\mathcal{N}_i^v} b_{ij}(v_j - v_i) + C_ie_i.\end{aligned} \tag{23}$$

Similar to the derivation of eq. (11), from eq. (8), one can obtain that

$$\begin{aligned}\dot{\bar{x}}_f &= \bar{v}_f,\\ \dot{\bar{v}}_f &= -(L_{p1}\otimes I_m)\bar{x}_f - (L_{v1}\otimes I_m)\bar{v}_f + Ce.\end{aligned} \tag{24}$$

Then, we have the following result.

Theorem 2: Suppose that Assumption 1 holds, G^p and G^v both contain a united spanning tree, and G_f^p is undirected. Then, the containment control problem for multi agent system consists of (1) and (2) with the disturbances are generated from the nonlinear exosystems (17) is solved by the control protocol (22) based on the nonlinear disturbance observer (18) if the matrices $A_i - K_iC_i, i \in \mathbb{F}$ are Hurwitz and the matrix $L_v + L_v^T$ is positive definite.

Proof: Consider the system consists of (19) and (24), i.e.,

$$\begin{aligned}\dot{\bar{x}}_f &= \bar{v}_f,\\ \dot{\bar{v}}_f &= -(L_{p1}\otimes I_m)\bar{x}_f - (L_{v1}\otimes I_m)\bar{v}_f + Ce,\\ \dot{e}_i &= (A_i - K_iC_i)e_i + \phi_i(\xi_i) - \phi_i(\hat{\xi}_i) - Q_i^{-1}\zeta_ie_i.\end{aligned} \tag{25}$$

Consider the following Lyapunov function candidate

$$V(t) = \frac{1}{2}\bar{x}_f^T(L_{p1}\otimes I_m)\bar{x}_f + \frac{1}{2}\bar{v}_f^T\bar{v}_f + \sum_{i=1}^{N} V_i, \tag{26}$$

where V_i is the Lyapunov function defined by Lemma 2, which satisfying the inequality (21).

The derivative of $V(t)$ along the trajectories of (25) gives

$$\dot{V}(t) \leq -\frac{\lambda_{min}(\frac{L_v+L_v^T}{2})}{2}\|\bar{v}_f\|^2 - \frac{\varepsilon}{2}\|e\|^2. \tag{27}$$

Thus, from (27), it follows that $V(t)$ is bounded, so are $\bar{x}_f, \bar{v}_f$, and e. Finally, similar to the proof of Theorem 1, and by using the LaSalle's invariance principle [8], it is easy to see that $\bar{x}_f \to 0$ and $\bar{v}_f \to 0$ as $t \to \infty$, which means that the containment control problem is solved. ■

References

[1] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.
[2] Y. Hong, J. Hu, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, 2006.
[3] M. Ji, G. Ferraritrecate, M. Egerstedt, and A. Buffa, "Containment control in mobile networks," *IEEE Transactions on Automatic Control*, vol. 53, no. 8, pp. 1972–1975, 2008.
[4] L. Rong, S. Xu, B. Zhang, and Y. Zou, "Accelerating average consensus by using the information of second-order neighbours with communication delays," *International Journal of Systems Science*, vol. 44, no. 6, pp. 1181–1188, 2013.
[5] P. Wang and Y. Jia, "Adaptive containment control of second-order multi-agent systems with nonlinear dynamics and multiple input-bounded leaders," *International Journal of Systems Science*, vol. 46, no. 13, pp. 2303–2316, 2015.
[6] J. Qin, W. X. Zheng, H. Gao, Q. Ma, and W. Fu, "Containment control for second-order multiagent systems communicating over heterogeneous networks." *IEEE Transactions on Neural Networks & Learning Systems*, vol. 28, no. 9, pp. 2143–2155, 2016.
[7] D. Xu and J. Huang, "Robust adaptive control of a class of nonlinear systems and its applications," *IEEE Transactions on Circuits & Systems I Regular Papers*, vol. 57, no. 3, pp. 691–702, 2010.
[8] H. K. Khalil, "Nonlinear systems third edition," *Upper Saddle River, NJ: Prentice-Hall, Inc.*, 2002.