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Distributed fault detection observer design for linear systems

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***Abstract*— This paper investigates the distributed fault detection problem for linear time-invariant (LTI) systems with distributed measurement output. We propose a distributed fault detection observer (DFDO) design method to detect actuator faults of the monitored system in the presence of a bounded process disturbance. The DFDO consists of a network of local fault detection observers, which communicate with their neighbors as prescribed by the given network graph. A systematic algorithm for DFEO design is addressed, enabling the residual to be robust against the effects of the external bounded process disturbance. Based on L_∞ analysis, a bank of linear matrix inequalities is presented to calculate the gain matrices and residual thresholds in our distributed fault detection scheme. Finally, we illustrate the effectiveness of the proposed distributed fault detection approach by means of a numerical simulation.**

Keywords: Distributed fault detection, linear system observers, LMIs, sensor networks.

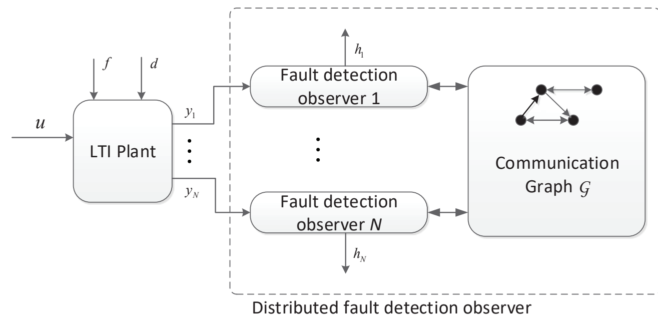

Fig. 1. Framework of distributed fault detection observer

I. INTRODUCTION

In the past three decades, fault detection and isolation (FDI) have been extensively studied to improve the reliability of modern control systems (see, e.g., [1], [2], [3], [4] and the references therein). Model-based fault detection has attracted considerable attention and numerous results have been reported [5], [6], [7], [8], [9]. Among the model-based fault detection schemes, observer-based fault detection is well-established and plays an important role in research and application domains. However, most of the existing FDI methods developed up to now assume that measurement outputs are obtained from sensors that are centrally located.

As the size and complexity of systems increase, several practical systems are large-scale and/or physically output distributed. For these systems, some fault diagnosis approaches have been proposed in the literature. For example, in [10], a robust centralized fault estimation method based on the sliding mode observer technique was proposed for multi-agent system exchanging relative information. Considering probabilistic performance, an FDI filter was designed for high dimensional nonlinear systems in [11]. We note that the fault diagnosis and fault estimation schemes proposed in the above literature are still in a centralized form. Some research on decentralized or distributed FDI was carried out in the literature as well [12], [13], [14]. In [15] fault tolerant decentralized H_∞ control for symmetric composite systems was presented. In [16], a decentralized FDI scheme was studied for a network system. A multi-layer distributed FDI scheme was proposed for large-scale systems in [17]. In addition, a distributed fault detection approach for interconnected second-order systems was studied in [18]. The monitored plant discussed in the above literature can be separated into several interconnected subsystems. Each fault filter or observer is designed for the corresponding subsystem. For large-scale systems that do not physically consist of some subsystems or can not be separated into several interconnected subsystems, distributed fault diagnosis was studied only in few publications. For a single monitored discrete-time system, a distributed fault diagnosis algorithm was proposed by using average-consensus techniques in [19].

Motivated by the above, this paper studies the distributed fault detection problem for continuous-time linear time invariant (LTI) systems with actuator faults. The measured output of the original plant is physically distributed and the proposed distributed fault detection observer (DFDO) consists of a network of local fault detection observers with a priori given network graph (see Fig. 1 for an illustration). Each local fault detection observer has access to only a portion of the output of the known monitored system, and communicates with its neighboring fault detection observers. The local fault detection observer at each node is designed to generate a residual which is robust against process disturbances. The gain matrices in the DFDO are obtained by solving linear matrix inequalities (LMI's). In this paper, the residual generation and residual threshold calculation are integrated together by using L_∞ analysis.

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II. Preliminaries and Problem Formulation

A. Preliminaries

Notation: For a given matrix M, its transpose is denoted by M^T and M^{-1} denotes its inverse. The symmetric part of a square real matrix M is sometimes denoted by $\text{Sym}(M) := M+M^T$. The rank of the matrix M is denoted by rank M. The identity matrix of dimension N will be denoted by I_N. The vector $\mathbf{1}_N$ denotes the $N \times 1$ column vector comprising of all ones. For a symmetric matrix P, $P > 0$ ($P < 0$) means that P is positive (negative) definite. For a set $\{A_1, A_2, \cdots, A_N\}$ of matrices, we use $\text{diag}\{A_1, A_2, \cdots, A_N\}$ to denote the block diagonal matrix with the A_i's along the diagonal, and the matrix $\begin{bmatrix} A_1^T & A_2^T & \cdots & A_N^T \end{bmatrix}^T$ is denoted by $\text{col}(A_1, A_2, \cdots, A_N)$. The Kronecker product of the matrices M_1 and M_2 is denoted by $M_1 \otimes M_2$. For a linear map $A : \mathscr{X} \to \mathscr{Y}$, ker $A := \{x \in \mathscr{X} | Ax = 0\}$ and im $A := \{Ax | x \in \mathscr{X}\}$ will denote the kernel and image of A, respectively. For a real inner product space $\mathscr{X}$, if $\mathscr{V}$ is a subspace of $\mathscr{X}$, then $\mathscr{V}^{\perp}$ will denote the orthogonal complement of $\mathscr{V}$. For a signal $x(t) \in \mathbb{R}^n$, its L_∞ norm is defined as $\|x\|_\infty = \sup_{t \geqslant 0} \|x(t)\|$, where $\|x(t)\|$ denotes the Euclidean norm of $x(t)$, i.e. $\|x(t)\| = \sqrt{x^T(t)x(t)}$.

In this paper, a weighted directed graph is denoted by $\mathscr{G} = (\mathscr{N}, \mathscr{E}, \mathscr{A})$, where $\mathscr{N} = \{1, 2, \cdots, N\}$ is a finite nonempty set of nodes, $\mathscr{E} \subset \mathscr{N} \times \mathscr{N}$ is an edge set of ordered pairs of nodes, and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ denotes the adjacency matrix. The (j,i)-th entry a_{ji} is the weight associated with the edge (i, j). We have $a_{ji} \neq 0$ if and only if $(i, j) \in \mathscr{E}$. Otherwise $a_{ji} = 0$. An edge $(i, j) \in \mathscr{E}$ designates that the information flows from node i to node j. A graph is said to be undirected if it has the property that $(i, j) \in \mathscr{E}$ implies $(j,i) \in \mathscr{E}$ for all $i, j \in \mathscr{N}$. We will assume that the graph is simple, i.e., $a_{ii} = 0$ for all $i \in \mathscr{N}$. For an edge (i, j), node i is called the parent node, node j the child node and j is a neighbor of i. A directed path from node i_1 to i_l is a sequence of edges (i_k, i_{k+1}), $k = 1, 2, \cdots, l-1$ in the graph. A directed graph $\mathscr{G}$ is strongly connected if between any pair of distinct nodes i and j in $\mathscr{G}$, there exists a directed path from i to j, $i, j \in \mathscr{N}$.

The Laplacian $\mathscr{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathscr{G}$ is defined as $\mathscr{L} := \mathscr{D} - \mathscr{A}$, where the i-th diagonal entry of the diagonal matrix $\mathscr{D}$ is given by $d_i = \sum_{j=1}^{N} a_{ij}$. By construction, $\mathscr{L}$ has a zero eigenvalue with a corresponding eigenvector $\mathbf{1}_N$ (i.e., $\mathscr{L}\mathbf{1}_N = 0_N$), and if the graph is strongly connected, all the other eigenvalues lie in the open right-half complex plane.

For strongly connected graphs $\mathscr{G}$, we review the following lemma.

Lemma 1. [20], [21], [22] Assume $\mathscr{G}$ is a strongly connected directed graph. Then there exists a unique positive row vector $r = \begin{bmatrix} r_1, \cdots, r_N \end{bmatrix}$ such that $r\mathscr{L} = 0$ and $r\mathbf{1}_N = N$. Define $R := \text{diag}\{r_1, \cdots, r_N\}$. Then $\hat{\mathscr{L}} := R\mathscr{L} + \mathscr{L}^T R$ is positive semi-definite, $\mathbf{1}_N^T \hat{\mathscr{L}} = 0$ and $\hat{\mathscr{L}}\mathbf{1}_N = 0$.

We note that $R\mathscr{L}$ is the Laplacian of the balanced digraph obtained by adjusting the weights in the original graph. The matrix $\hat{\mathscr{L}}$ is the Laplacian of the undirected graph obtained by taking the union of the edges and their reversed edges in this balanced digraph. This undirected graph is called the mirror of this balanced graph [20].

B. Problem formulation

In this paper, we consider a continuous-time LTI system subject to actuator faults and disturbances represented by

$$\begin{cases} \dot{x} = Ax + Bu + Ff + Ed \\ y = Cx \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^r$ is the input, $f \in \mathbb{R}^q$ is the fault, $d \in \mathbb{R}^l$ is the disturbance, and $y \in \mathbb{R}^m$ is the measurement output. $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times r}, F \in \mathbb{R}^{n \times q}, E \in \mathbb{R}^{n \times l}, C \in \mathbb{R}^{m \times n}$ are known constant matrices with appropriate dimensions. We assume that d is unknown but bounded, and that $\|d\|_\infty$ is a known constant. We partition the output y as $y = \text{col}(y_1, \cdots, y_N)$, where $y_i \in \mathbb{R}^{m_i}$ and $\sum_{i=1}^{N} m_i = m$. Accordingly, $C = \text{col}(C_1, \cdots, C_N)$ with $C_i \in \mathbb{R}^{m_i \times n}$. Here, the portion $y_i = C_i x \in \mathbb{R}^{m_i}$ is assumed to be the only information that can be acquired by node i in the DFDO.

In this paper, a standing assumption will be that the communication graph is a strongly connected directed graph. We will also assume that the pair (C, A) is observable. However, (C_i, A) is not necessarily observable or detectable.

We will design a DFDO for the system given by (1) with the given communication network. The DFDO will consist of N local fault detection observers, and the local fault detection observer at node i has the following dynamics

$$\begin{cases} \dot{\hat{x}}_i = & A\hat{x}_i + L_i(y_i - C_i\hat{x}_i) + Bu \\ & +\gamma r_i M_i \sum_{j=1}^{N} a_{ij}(\hat{x}_j - \hat{x}_i) \quad, \ i \in \mathscr{N} \\ h_i = & y_i - C_i\hat{x}_i \end{cases} \tag{2}$$

where $\hat{x}_i \in \mathbb{R}^n$ is the state of the local observer at node i, $h_i \in \mathbb{R}_i^m$ is the residual of the local fault detection observer at node i, a_{ij} is the (i, j)-th entry of the adjacency matrix $\mathscr{A}$ of the given network, r_i is defined as in Lemma 1, $\gamma \in \mathbb{R}$ is a coupling gain to be designed, and $L_i \in \mathbb{R}^{n \times m_i}$ and $M_i \in \mathbb{R}^{n \times n}$ are gain matrices to be designed.

To analyze and synthesize observer (2), we define the local estimation error of the i-th observer as

$$e_i := \hat{x}_i - x. \tag{3}$$

By combining (1) and (2) we find that the error of the i-th local fault detection observer is represented by

$$\begin{cases} \dot{e}_i = & (A - L_iC_i)e_i - Ed - Ff \\ & +\gamma r_i M_i \sum_{j=1}^{N} a_{ij}(e_j - e_i) \quad, \ i \in \mathscr{N}. \\ h_i = & C_i e_i \end{cases} \tag{4}$$

Let $e := \text{col}(e_1, e_2, \cdots, e_N)$ be the joint vector of errors and $\tilde{d} := \mathbf{1}_N \otimes d$ be the extended disturbance vector. Then we obtain the global error system

$$\begin{cases} \dot{e} = \Lambda e - \gamma M(R\mathscr{L} \otimes I_n)e - \tilde{E}\tilde{d} - \tilde{F}f, \\ h_i = C_i e_i, \ i \in \mathscr{N}. \end{cases} \tag{5}$$

where

$$\Lambda = \text{diag}\{A - L_1C_1, \cdots, A - L_NC_N\},$$

$$M = \text{diag}\{M_1, \cdots, M_N\},$$

$$\tilde{E} = I_N \otimes E, \ \tilde{F} = \mathbf{1}_N \otimes F,$$

and R is as defined in Lemma 1. It is noted that $\tilde{d}$ is bounded since d is bounded.

Here, we will discuss how to design gain matrices for the DFDO (2) so that error system (5) is internally stable while attenuating the effect of the extended disturbance signal on the residual. More specifically, we want to design a DFDO such that the following specifications hold:

(i) The error system (5) is internally stable, i.e., it is asymptotically stable if the extended disturbance vector $\tilde{d}$ and the fault f are zero.

(ii) In fault-free condition, the error system (5) satisfies a given L_∞ performance level $\beta_i > 0, i \in \mathcal{N}$, i.e., fot all $t \geqslant 0$

$$\|h_i(t)\| \leqslant \beta_i \sqrt{V(0)\mathrm{e}^{-\alpha t} + N\|d\|_\infty^2} \tag{6}$$

where $V(0) = e(0)^T Pe(0), P > 0$ is a positive definite matrix to be specified, $\alpha > 0$ is a given positive scalar and N is the number of nodes.

Since (C_i, A) is not necessarily observable or detectable, L_i cannot be designed using any classical method directly. We use an orthogonal transformation that yields an observability decomposition for the pair (C_i, A). For $i \in \mathcal{N}$, let T_i be an orthogonal matrix, i.e., a square matrix such that $T_i T_i^T = I_n$, such that the matrices A and C_i are transformed by the state space transformation T_i into the form

$$T_i^T A T_i = \begin{bmatrix} A_{io} & 0 \\ A_{ir} & A_{iu} \end{bmatrix},\ C_i T_i = \begin{bmatrix} C_{io} & 0 \end{bmatrix},\ T_i^T E = \begin{bmatrix} E_{io} \\ E_{iu} \end{bmatrix} \tag{7}$$

where $C_{io} \in \mathbb{R}^{p_i \times v_i}$, $A_{io} \in \mathbb{R}^{v_i \times v_i}$, $A_{ir} \in \mathbb{R}^{(n-v_i)\times v_i}$, $A_{iu} \in \mathbb{R}^{(n-v_i)\times(n-v_i)}$, and $n - v_i$ is the dimension of the unobservable subspace of the pair (C_i, A). Clearly, by construction, the pair (C_{io}, A_{io}) is observable. In addition, if we partition $T_i = \begin{bmatrix} T_{i1} & T_{i2} \end{bmatrix}$, where T_{i1} consists of the first v_i columns of T_i, then the unobservable subspace is given by im $T_{i2} = \ker O_i$, where $O_i = \mathrm{col}(C_i, C_i A, \cdots, C_i A^{n-1})$. Note that im $T_{i1} = (\ker O_i)^\perp$.

III. Main Results

A. Distributed fault detection observer design

In this part, we study the DFDO design. Before presenting the main design procedure, we state the following lemmas based on Lemma 1. Our first lemma is as follows:

Lemma 2.[23] For a strongly connected directed graph $\mathcal{G}$, zero is a simple eigenvalue of $\hat{\mathcal{L}} = R\mathcal{L} + \mathcal{L}^T R$ introduced in Lemma 1. Furthermore, its eigenvalues can be ordered as $\lambda_1 = 0 < \lambda_2 \leqslant \lambda_3 \leqslant \cdots \leqslant \lambda_N$. Furthermore, there exists an orthogonal matrix $U = \begin{bmatrix} \frac{1}{\sqrt{N}}\mathbf{1}_N & U_2 \end{bmatrix}$, where $U_2 \in \mathbb{R}^{N\times(N-1)}$, such that $U^T(R\mathcal{L} + \mathcal{L}^T R)U = \mathrm{diag}\{0, \lambda_2, \cdots, \lambda_N\}$.

Our second lemma was proven in [24]. The statement of the lemma is as follows:

Lemma 3. Let $\mathcal{L}$ be the Laplacian matrix associated with the strongly connected directed graph $\mathcal{G}$. For all $g_i > 0$, $i \in \mathcal{N}$, there exists $\varepsilon > 0$ such that

$$T^T(\hat{\mathcal{L}} \otimes I_n)T + G > \varepsilon I_{nN}, \tag{8}$$

where $T = \mathrm{diag}\{T_1, \cdots, T_N\}$, $\hat{\mathcal{L}}$ is defined as in Lemma 1, $G = \mathrm{diag}\{G_1, \cdots, G_N\}$, and $G_i = \begin{bmatrix} g_i I_{v_i} & 0 \\ 0 & 0_{n-v_i} \end{bmatrix}$, $i \in \mathcal{N}$.

The following theorem now deals with the existence of a DFDO of the form (2) that satisfies (i) and (ii). A condition for its existence is expressed in terms of solvability of an LMI. Solutions to the LMIs yield required gain matrices. Let $r_i > 0$, $i \in \mathcal{N}$, be as in Lemma 1. Let $g_i > 0$, $i \in \mathcal{N}$, and $\varepsilon > 0$ be such that (8) holds. Finally, let $\gamma \in \mathbb{R}$. We have the following:

Theorem 4 Given $\alpha > 0$ and $\beta_i > 0$, there exist gain matrices L_i and M_i, $i \in \mathcal{N}$, such that the DFDO (2) satisfies the specifications (i) and (ii) if there exist a positive scalar $\gamma > 0$ and positive definite matrices $P_{io} \in \mathbb{R}^{v_i \times v_i}, P_{io} > 0$, $P_{iu} \in \mathbb{R}^{(n-v_i)\times(n-v_i)}, P_{iu} > 0$, and a matrix $W_i \in \mathbb{R}^{v_i \times p_i}$ such that

$$\begin{bmatrix} \Psi_{11i} & \Psi_{12i} & \Psi_{13i} \\ \star & \Psi_{22i} & \Psi_{23i} \\ \star & \star & \Psi_{33i} \end{bmatrix} < 0,\ \forall i \in \mathcal{N}, \tag{9}$$

$$C_{io}^T C_{io} - \beta_i^2 P_{io} < 0 \tag{10}$$

where

$$\begin{aligned} \Psi_{11i} &= P_{io}A_{io} + A_{io}^T P_{io} - W_i C_{io} - C_{io}^T W_i^T + \alpha P_{io} + \gamma g_i I_{v_i} \\ &\quad - \gamma\varepsilon I_{v_i}, \\ \Psi_{12i} &= A_{ir}^T P_{iu}, \\ \Psi_{13i} &= P_{io}E_{io}, \\ \Psi_{22i} &= \mathrm{Sym}(P_{iu}A_{iu}) - \gamma\varepsilon I_{n_q - v_i} + \alpha P_{iu}, \\ \Psi_{23i} &= P_{iu}E_{iu}, \\ \Psi_{33i} &= -\alpha_i I_l, \end{aligned}$$

and E_{io}, E_{iu} are defined in (7). In that case, the gain matrices in the distributed observer (2) can be taken as

$$L_i := T_i \begin{bmatrix} L_{io} \\ 0 \end{bmatrix},\ M_i := T_i \begin{bmatrix} \mathscr{P}_{io}^{-1} & 0 \\ 0 & P_{iu}^{-1} \end{bmatrix} T_i^T, \tag{11}$$

where $L_{io} = P_{io}^{-1} W_i$, $i \in \mathcal{N}$.

Proof: Choose a candidate Lyapunov function for the error system (5)

$$V(e_1, \cdots, e_N) := \sum_{i=1}^N e_i^T P_i e_i, \tag{12}$$

where $P_i := T_i \begin{bmatrix} P_{io} & 0 \\ 0 & P_{iu} \end{bmatrix} T_i^T$. Clearly then $P_i > 0$.

The time-derivative of V is

$$\begin{aligned} \dot{V}(e) =& e^T(P\Lambda + \Lambda^T P)e + e^T P\tilde{E}\tilde{d} + \tilde{d}^T \tilde{E}^T Pe \\ & - \gamma e^T(P\overline{M}(R\mathcal{L} \otimes I_n) + (\mathcal{L}^T R \otimes I_n)\overline{M}^T P)e \end{aligned} \tag{13}$$

where $P = \mathrm{diag}\{P_1, \cdots, P_N\}$. Since the matrix M_i in (11) is chosen as $M_i = P_i^{-1}$, we have $\overline{M} = P^{-1}$. Hence, the time-derivative of V becomes

$$\dot{V}(e) = e^T(P\Lambda + \Lambda^T P - \gamma\hat{\mathcal{L}} \otimes I_n)e + e^T P\tilde{E}\tilde{d} + \tilde{d}^T \tilde{E}^T Pe, \tag{14}$$

where, as before, $\hat{\mathcal{L}} = R\mathcal{L} + \mathcal{L}^T R$.

On the other hand, from (9) and (8) in Lemma 3, we obtain

$$\begin{bmatrix} \mathrm{diag}\{\mathcal{Q}_1, \cdots, \mathcal{Q}_N\} - T^T\gamma(\hat{\mathcal{L}} \otimes I_n)T & T^T P\tilde{E} \\ \tilde{E}^T PT & -\alpha I_{Nl} \end{bmatrix} < 0, \tag{15}$$

where
$$\mathscr{Q}_i = \begin{bmatrix} \Phi_i & A_{ir}^T P_{iu} \\ P_{iu}A_{ir} & P_{iu}A_{iu} + A_{iu}^T P_{iu} + 2\alpha P_{iu} \end{bmatrix}, \ i \in \mathscr{N},$$
with $\Phi_i := P_{io}A_{io} + A_{io}^T P_{io} - W_i C_{io} - C_{io}^T W_i^T + \alpha P_{io}$.

By taking $L_{io} = P_{io}^{-1} W_i$ and pre- and post- multiplying the inequality (15) with $\mathrm{diag}\{T, I_{Nl_q}\}$ and its transpose, we get

$$\begin{bmatrix} P\Lambda + \Lambda^T P - \gamma \hat{\mathscr{L}} \otimes I_n & P\tilde{E} \\ \tilde{E}^T P & -\alpha I_{Nl_q} \end{bmatrix} < 0, \tag{16}$$

By pre- and post- multiplying the inequality (16) with $[e^T \ \tilde{d}^T]$ and its transpose, we have

$$\dot{V}(e) \leqslant -\alpha V(e) + \alpha \tilde{d}^T(t)\tilde{d}(t) \tag{17}$$

which implies that

$$\begin{aligned} V(e(t)) &\leqslant V(0)\mathrm{e}^{-\alpha t} + \alpha \|\tilde{d}\|_\infty^2 \int_0^t \mathrm{e}^{-\alpha(t-\tau)} \mathrm{d}\tau \\ &\leqslant V(0)\mathrm{e}^{-\alpha t} + (1 - \mathrm{e}^{-\alpha t}) N \|d\|_\infty^2 \\ &\leqslant V(0)\mathrm{e}^{-\alpha t} + N\|d\|_\infty^2 \end{aligned} \tag{18}$$

where $V(0) = e^T(0)Pe(0)$.

From (10), we have

$$C_i^T C_i - \beta_i^2 P_i < 0 \tag{19}$$

which implies that

$$\begin{aligned} \|h_i(t)\|^2 &\leqslant \beta_i^2 e_i^T(t) P_i e_i(t) \\ &\leqslant \beta_i^2 e^T(t) P e(t) \\ &\leqslant \beta_i^2 (V(0)\mathrm{e}^{-\alpha t} + N\|d\|_\infty^2) \end{aligned} \tag{20}$$

That is, L_∞ performance index (6) is satisfied. Therefore, conditions (i) and (ii) are both satisfied.

■

B. Distributed fault detection scheme

For the residual evaluation, one of the commonly used approaches is the so-called threshold method [2]. In this paper, we adopt the following logical relationship for fault detection

$$\begin{aligned} H_i(t) \le H_{thi}(t)\ ,\forall i \in \mathscr{N} &\implies \text{fault free} \\ H_i(t) > H_{thi}(t)\ ,\exists i \in \mathscr{N} &\implies \text{fault occurs} \end{aligned} \tag{21}$$

where the residual evaluation function at each node is defined as the 2-norm of the vector h_i, namely $H_i(t) = \|h_i(t)\|$. Different from the widely-used constant threshold, a time-varying threshold is obtained by L_∞ analysis. Therefore we adopt the following time-varying threshold

$$H_{thi}(t) = \beta_i \sqrt{\lambda_{\max} \bar{e}_0^2 \mathrm{e}^{-\alpha t} + N\|d\|_\infty^2}$$

where $\bar{e}_0 \in \mathbb{R}$ denotes the upper bound of $\|e(0)\|$, $\lambda_{\max}$ is the maximum eigenvalue of $P \in \mathbb{R}^{n_x \times n_x}$, $P > 0$ which is obtained by Theorem 4.

Based on the previous lemmas and theorem we have the following result:
Let $\alpha > 0$. We assume that (C, A) is observable and $\mathscr{G}$ is a strongly connected directed graph, then a DFDO (2) that detects faults and attenuates the effect of the disturbance is designed using the following algorithm.

Algorithm 1 Distributed fault detection

1: For each $i \in \mathscr{N}$, choose an orthogonal matrix T_i such that
$$T_i^T A T_i = \begin{bmatrix} A_{io} & 0 \\ A_{ir} & A_{iu} \end{bmatrix}, C_i T_i = \begin{bmatrix} C_{io} & 0 \end{bmatrix}, T_i^T E = \begin{bmatrix} E_{io} \\ E_{iu} \end{bmatrix}$$
with (C_{io}, A_{io}) observable.
2: Compute the positive row vector $r = \begin{bmatrix} r_1, \cdots, r_N \end{bmatrix}$ such that $r\mathscr{L} = 0$ and $r\mathbf{1}_N = N$.
3: Solve the LMI (8) and get $g_i,\ i \in \mathscr{N}$ and ε.
4: Solve the LMI's (9) and (10) for all $i \in \mathscr{N}$ and get γ, $P_{io},\ P_{iu},\ W_i, \beta_i$.
5: Define
$$L_i := T_i \begin{bmatrix} P_{io}^{-1} W_i \\ 0 \end{bmatrix}, M_i := T_i \begin{bmatrix} P_{io}^{-1} & 0 \\ 0 & P_{iu}^{-1} \end{bmatrix} T_i^T, i \in \mathscr{N}$$
6: Calculate the local residual signal h_i at each node i using local fault detection observer (2).
7: Calculate the local time-varying threshold H_{thi}.
8: Make the fault detection decision by comparing the residual evaluation function $H_i(t)$ with time-varying threshold $H_{thi}(t)$ at each node i.

Remark 5: In the special case that the communication graph among the observers is a connected undirected graph, we have that $r = \mathbf{1}_N^T$ is the unique positive row vector such that $r\mathscr{L} = 0$ and $r\mathbf{1}_N = N$. In the design procedure of Algorithm 1, we can then take $r_i = 1$ for all $i \in \mathscr{N}$.

IV. Simulation Example

In this section, we will use a numerical example borrowed from [25] to illustrate the effectiveness of our approach.

Consider a linear system (1) with coefficient matrices given by

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & -2 & -1 & -1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ -8 & 1 & -1 & -1 & -2 & 0 \\ 4 & -0.5 & 0.5 & 0 & 0 & -4 \end{bmatrix}, \ B = F = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{bmatrix},$$

$$C = \left[\begin{array}{cccccc} 1 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ \hline 2 & 0 & 1 & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & 2 & 0 & 0 \\ \hline 1 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 4 & 0 & 0 & 0 \end{array}\right] = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}, \ E = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

The communication network is given by the strongly connected digraph in Fig. 2. The Laplacian of this graph

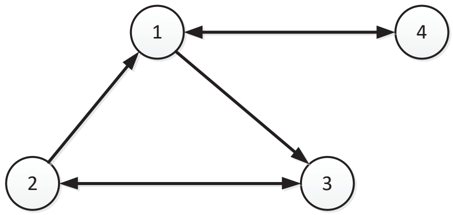

Fig. 2. The communication graph among nodes

is given by

$$\mathscr{L} = \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

It can be seen that none of the local systems (C_i, A) is observable, but (C, A) is an observable pair. We will apply the conceptual Algorithm 1 to design a distributed observer. The normalized positive left eigenvector of the Laplacian is computed to be $r = \begin{bmatrix} 0.8 & 1.6 & 0.8 & 0.8 \end{bmatrix}$.

We choose $\alpha = 8$, $\beta_1 = 0.0497$, $\beta_2 = 0.0346$, $\beta_3 = 0.0387$ and $\beta_4 = 0.0648$. Following Algorithm 1, a coupling gain is computed to be $\gamma = 0.6087$. The local observer gain matrices are computed as:

$$L_1 = \begin{bmatrix} -6.3380 & 3.7506 \\ 0 & 0 \\ 0 & 0 \\ 0.6528 & 5.6815 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \; L_2 = \begin{bmatrix} 312.5684 \\ -736.9605 \\ -900.7698 \\ 808.9113 \\ -943.6320 \\ 303.3418 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2.0023 \\ 0 \\ 0 \end{bmatrix}, \; L_4 = \begin{bmatrix} 71.7864 & -0.0000 \\ -670.3341 & 0.0000 \\ -25.2502 & 0.0000 \\ 678.2653 & -0.0000 \\ -632.6460 & 0.0000 \\ 144.5728 & -0.0000 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 0.0002 & 0 & 0 & -0.0001 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -0.0001 & 0 & 0 & 0.0003 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.1900 & -0.1974 & -0.5356 & 0.2344 & -0.4143 & 0.1628 \\ -0.1974 & 0.7994 & 0.6001 & -1.0203 & 0.7559 & -0.2225 \\ -0.5356 & 0.6001 & 1.5137 & -0.7218 & 1.1901 & -0.4631 \\ 0.2344 & -1.0203 & -0.7218 & 1.4085 & -0.8829 & 0.2714 \\ -0.4143 & 0.7559 & 1.1901 & -0.8829 & 1.1172 & -0.3836 \\ 0.1628 & -0.2225 & -0.4631 & 0.2714 & -0.3836 & 0.1445 \end{bmatrix},$$

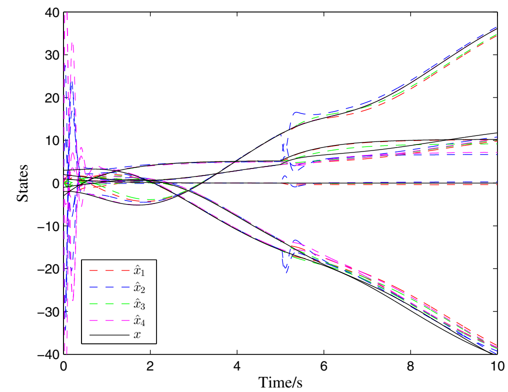

Fig. 3. The state components of the observed plant and their estimates

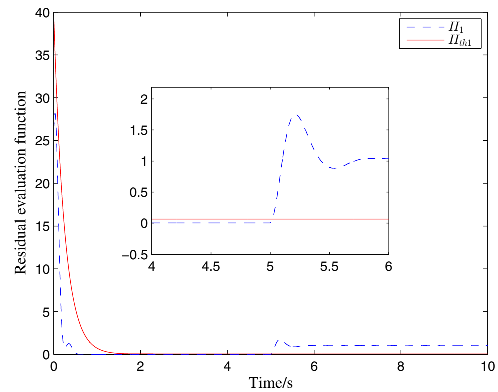

Fig. 4. The residual evaluation function and its threshold at node 1

$$M_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0002 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} 0.0502 & -0.0529 & -0.0245 & -0.0264 & -0.2237 & 0.0635 \\ -0.0529 & 0.6743 & 0.0204 & -0.7865 & 0.5335 & -0.1191 \\ -0.0245 & 0.0204 & 0.0121 & 0.0191 & 0.1062 & -0.0305 \\ -0.0264 & -0.7865 & 0.0191 & 1.1171 & -0.2568 & 0.0334 \\ -0.2237 & 0.5335 & 0.1062 & -0.2568 & 1.1523 & -0.3097 \\ 0.0635 & -0.1191 & -0.0305 & 0.0334 & -0.3097 & 0.0852 \end{bmatrix}.$$

For our simulation, the disturbance is chosen as random noise with bound $\|d\|_\infty = 0.1$. In addition, we take the following actuator fault:

$$f(t) = \begin{cases} 0 & 0 \leqslant t < 5 \\ 5 & 5 \leqslant t \leqslant 10 \end{cases} \tag{22}$$

where the time units are seconds.

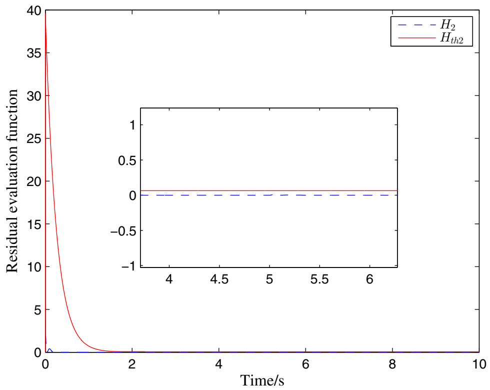

Fig. 5. The residual evaluation function and its threshold at node 2

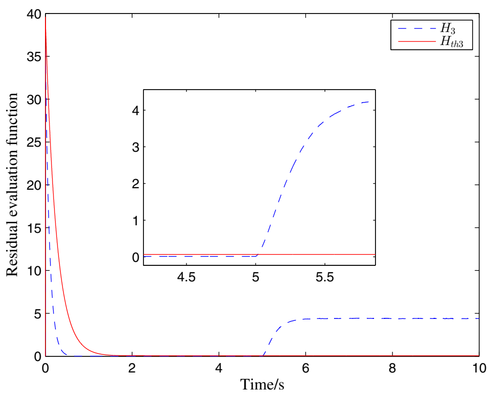

Fig. 6. The residual evaluation function and its threshold at node 3

In the simulation, the initial state of the observed system is taken as $x(0) = \begin{bmatrix} 1 & 3 & -2 & -3 & -1 & 2 \end{bmatrix}^T$. For each local fault detection observer the initial state is taken to be zero.

The state components and their estimates are depicted in Fig. 3. It can be seen that all estimates converge to the actual state components before the fault occurring. Each local fault detection observer does not track the real state when the actuator has a fault. Figs. 4-7 show the residual evaluation functions and their time-varying thresholds associated with each local fault detection observer. It can be seen that the residual evaluation functions at nodes 1 and 3 exceed their thresholds when the fault occurs.

V. Conclusions

In this paper, we have presented a distributed observer-based fault detection scheme for LTI systems with a bounded process disturbance. A network of local fault detection observers are built at each measurement node. The information among the local fault detection observers is exchanged by a known strongly connected directed graph. The local fault detection observer at each node is designed to detect the actuator fault of the monitored system. By using L_∞ analysis,

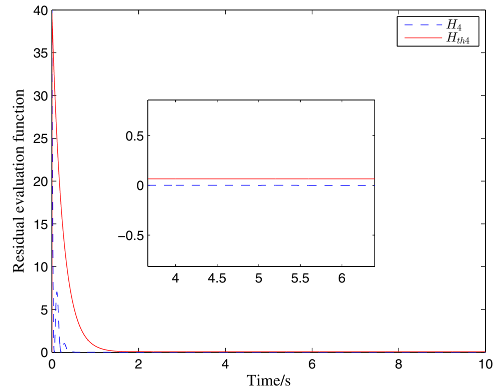

Fig. 7. The residual evaluation function and its threshold at node 4

a bank of LMI's is presented to calculate the gain matrices and residual thresholds in our DFDO. Finally, we have presented a simple algorithm to design a DFDO that achieves fault detection. In future research, we plan to focus on distributed fault isolation and accommodation.

References

[1] R. Isermann, *Fault-diagnosis systems: an introduction from fault detection to fault tolerance*. Springer Science & Business Media, 2006.

[2] S. Ding, *Model-based fault diagnosis techniques: design schemes, algorithms, and tools*. Springer Science & Business Media, 2008.

[3] J. Chen and R. J. Patton, *Robust model-based fault diagnosis for dynamic systems*. Springer Science & Business Media, 2012.

[4] K. Zhang, B. Jiang, and P. Shi, *Observer-based fault estimation and accomodation for dynamic systems*. Springer, 2012, vol. 436.

[5] M. Zhong, S. X. Ding, J. Lam, and H. Wang, "An LMI approach to design robust fault detection filter for uncertain lti systems," *Automatica*, vol. 39, no. 3, pp. 543–550, 2003.

[6] Z. Gao, C. Cecati, and S. X. Ding, "A survey of fault diagnosis and fault-tolerant techniquespart I: Fault diagnosis with model-based and signal-based approaches," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 6, pp. 3757–3767, 2015.

[7] H. Li, Y. Gao, P. Shi, and H.-K. Lam, "Observer-based fault detection for nonlinear systems with sensor fault and limited communication capacity," *IEEE Transactions on Automatic Control*, vol. 61, no. 9, pp. 2745–2751, 2016.

[8] C.-X. Shi, G.-H. Yang, and X.-J. Li, "Fault detection filter design with adaptive mechanism for linear uncertain polytopic systems in finite frequency domains," *IET Control Theory & Applications*, vol. 10, no. 16, pp. 2027–2037, 2016.

[9] Z. Wang, P. Shi, and C.-C. Lim, "H_-/H_∞ fault detection observer in finite frequency domain for linear parameter-varying descriptor systems," *Automatica*, vol. 86, pp. 38–45, 2017.

[10] P. P. Menon and C. Edwards, "Robust fault estimation using relative information in linear multi-agent networks," *IEEE Transactions on Automatic Control*, vol. 59, no. 2, pp. 477–482, 2014.

[11] P. M. Esfahani and J. Lygeros, "A tractable fault detection and isolation approach for nonlinear systems with probabilistic performance," *IEEE Transactions on Automatic Control*, vol. 61, no. 3, pp. 633–647, 2016.

[12] X. Zhang and Q. Zhang, "Distributed fault diagnosis in a class of interconnected nonlinear uncertain systems," *International Journal of Control*, vol. 85, no. 11, pp. 1644–1662, 2012.

[13] Y. Li, H. Fang, J. Chen, and C. Shang, "Distributed fault detection and isolation for multi-agent systems using relative information," in *Proc. American Control Conference (ACC)*, Boston, MA, USA, 2016, pp. 5939–5944.

[14] A. Marino, F. Pierri, and F. Arrichiello, "Distributed fault detection isolation and accommodation for homogeneous networked discrete-time linear systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 9, pp. 4840–4847, 2017.
[15] S. Huang, J. Lam, G.-H. Yang, and S. Zhang, "Fault tolerant decentralized H_∞ control for symmetric composite systems," *IEEE Transactions on Automatic Control*, vol. 44, no. 11, pp. 2108–2114, 1999.
[16] D. Sauter, T. Boukhobza, and F. Hamelin, "Decentralized and autonomous design for FDI/FTC of networked control systems," *IFAC Proceedings Volumes*, vol. 39, no. 13, pp. 138–143, 2006.
[17] F. Boem, R. M. Ferrari, C. Keliris, T. Parisini, and M. M. Polycarpou, "A distributed networked approach for fault detection of large-scale systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 1, pp. 18–33, 2017.
[18] I. Shames, A. M. Teixeira, H. Sandberg, and K. H. Johansson, "Distributed fault detection for interconnected second-order systems," *Automatica*, vol. 47, no. 12, pp. 2757–2764, 2011.
[19] E. Franco, R. Olfati-Saber, T. Parisini, and M. M. Polycarpou, "Distributed fault diagnosis using sensor networks and consensus-based filters," in *The 45th IEEE Conference on Decision and Control*, San Diego, CA, USA, 2006, pp. 386–391.
[20] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.
[21] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.
[22] W. Yu, G. Chen, M. Cao, and J. Kurths, "Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics," *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 40, no. 3, pp. 881–891, 2010.
[23] Z. Li and Z. Duan, *Cooperative control of multi-agent systems: a consensus region approach*. CRC Press, 2014.
[24] W. Han, H. L. Trentelman, Z. Wang, and Y. Shen, "A simple approach to distributed observer design for linear systems," *DOI: 10.1109/TAC.2018.2828103*, 2018.
[25] M. Saif and Y. Guan, "Decentralized state estimation in large-scale interconnected dynamical systems," *Automatica*, vol. 28, no. 1, pp. 215–219, 1992.