

Fundamental Limitations of Feedback and Networked Feedback Systems: An Information-Theoretic Analysis*

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Abstract—In this extended abstract we analyze the fundamental performance limitations of feedback and networked feedback systems by leveraging information-theoretic notions including entropy and mutual information, as an attempt towards bridging control theory and information theory. In particular, we develop Bode-type integrals, power gain bounds, and variance minimization limits applicable to information-constrained networked feedback systems, which hold for arbitrarily general controllers as long as they are causal and stabilizing. Towards this end, we propose a number of information measures such as negentropy rate, channel blurredness, and Gaussianity-whiteness compatible to control system analysis. We also investigate feedback systems without communication channels; in particular, we examine the results further in the context of state estimation. In general, the performance limitations are seen to be characterized by plant instabilities, channel noisiness, and disturbance Gaussianity-whiteness. In summary, this abstract presents a unifying information-theoretic framework for control system limitation analysis, and consolidates the role of Shannon’s information theory as a mathematical theory of not only communication but also control.

Keywords—Networked control systems, performance limitation, information theory, entropy, state estimation.

AMS subject classifications—93A99, 93E99, 94A99.

I. INTRODUCTION

Performance limitation [1], [2] is a topic of lasting interest in the study of feedback design, of which one canonical objective is to achieve desirable disturbance attenuation properties. For a linear time-invariant (LTI) system, disturbance attenuation at different frequencies can be characterized by the sensitivity function. With different performance indices in terms of the sensitivity function addressing different design goals, control performance tradeoffs and limits have been obtained, e.g., as Bode integrals and under the \mathcal{H}_∞ criterion.

Due to the various communication constraints present in the feedback loop, existing performance limitation results cannot be readily applied to networked feedback systems. The impact of communication constraints on control performance calls for the incorporation of information and communication constraints into performance limitation studies of feedback control [3].

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Towards this end, [4]–[7] developed Bode-type integrals using information-theoretic notions such as entropy and mutual information. In an attempt by the authors [8], [9], the Bode-type integrals were further strengthened for networked feedback systems by introducing new information measures, including negentropy rate and channel blurredness. We also utilized this framework to derive power gain bounds [9] and variance minimization limits [10]; for variance minimization, we proposed the notion of Gaussianity-whiteness to facilitate the analysis, and further examine the implications of the limits in the context of state estimation.

II. NOTATIONS AND BASIC CONCEPTS

Throughout this extended abstract, we consider real-valued continuous zero-mean random variables and vectors, as well as discrete-time stochastic processes. We denote random variables and vectors using boldface letters. The logarithm is defined with base 2, and all the functions are assumed to be measurable. The definitions and properties of asymptotically stationarity and asymptotic power spectrum $S_{\mathbf{x}}(\omega)$ can be found in [4], [11], while those of differential entropy $h(\mathbf{x})$, conditional entropy $h(\mathbf{x}|\mathbf{y})$, entropy rate $h_\infty(\mathbf{x})$, mutual information $I(\mathbf{x}; \mathbf{y})$, and mutual information rate $I_\infty(\mathbf{x}; \mathbf{y})$ can be found in [12].

Negentropy rate [9] is a measure of non-Gaussianity for asymptotically stationary processes, which generalizes the notion of negentropy to stochastic processes.

Definition 1: The negentropy of a random variable $\mathbf{x} \in \mathbb{R}$ with variance $\sigma_{\mathbf{x}}^2$ is defined as

$$J(\mathbf{x}) = \log \sqrt{2\pi e \sigma_{\mathbf{x}}^2} - h(\mathbf{x}).$$

The negentropy rate of an asymptotically stationary process $\{\mathbf{x}_k\}$, $\mathbf{x}_k \in \mathbb{R}$ with asymptotic power spectrum $S_{\mathbf{x}}(\omega)$ is defined as

$$J_\infty(\mathbf{x}) \triangleq \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \sqrt{2\pi e S_{\mathbf{x}}(\omega)} d\omega - h_\infty(\mathbf{x}). \quad (1)$$

It can be shown that $J_\infty(\mathbf{x}) \geq 0$, where the equality holds if and only if $\{\mathbf{x}_k\}$ is asymptotically Gaussian.

Spectral flatness [13] is an important notion for quantifying how flat the power spectral density of an asymptotically stationary process is, which also provides a measure of how white such a process is.

Definition 2: The spectral flatness of an asymptotically stationary process $\{\mathbf{x}_k\}$, $\mathbf{x}_k \in \mathbb{R}$ with asymptotic power spectrum $S_{\mathbf{x}}(\omega)$ is defined as

$$\gamma_{\mathbf{x}}^2 = \frac{2^{\frac{1}{2\pi}} \int_{-\pi}^{\pi} \log S_{\mathbf{x}}(\omega) d\omega}{\frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\mathbf{x}}(\omega) d\omega}. \quad (2)$$

It is known that $0 \leq \gamma_{\mathbf{x}}^2 \leq 1$, and that $\gamma_{\mathbf{x}}^2 = 1$ if and only if $\{\mathbf{x}_k\}$ is asymptotically white.

We now define Gaussianity-whiteness [3] by combining negentropy rate and spectral flatness in a non-trivial way, which provides a measure on how close an asymptotically stationary process is to being purely white Gaussian.

Definition 3: Consider an asymptotically stationary process $\{\mathbf{x}_k\}$, $\mathbf{x}_k \in \mathbb{R}$ with spectral flatness $\gamma_{\mathbf{x}}^2$ and negentropy rate $J_{\infty}(\mathbf{x})$. Its Gaussianity-whiteness is defined as

$$\text{GW}_{\mathbf{x}} \triangleq \left[2^{-2J_{\infty}(\mathbf{x})} \right] \gamma_{\mathbf{x}}^2. \quad (3)$$

Since $J_{\infty}(\mathbf{x}) \geq 0$ and $0 \leq \gamma_{\mathbf{x}}^2 \leq 1$, we have $0 \leq \text{GW}_{\mathbf{x}} \leq 1$. It is easy to verify that $\text{GW}_{\mathbf{x}} = 1$ if and only if $\{\mathbf{x}_k\}$ is asymptotically white Gaussian.

We next introduce the channel blurredness [8]. In contrast to channel capacity [12], channel blurredness is defined as the infimum of mutual information (rate) between the noise and output of a channel, and thus lends a more direct relationship between the channel noise and channel output.

Definition 4: Consider a general causal noisy channel with input process $\{\mathbf{v}_k\}$, noise process $\{\mathbf{n}_k\}$, and output process $\{\mathbf{u}_k\}$. Then, the blurredness of the channel, measured in bits, is defined as

$$B \triangleq \inf_{p_{\mathbf{v}}} I_{\infty}(\mathbf{n}; \mathbf{u}) = \inf_{p_{\mathbf{v}}} \limsup_{k \rightarrow \infty} \frac{I(\mathbf{n}_{0,\dots,k}; \mathbf{u}_{0,\dots,k})}{k+1}, \quad (4)$$

where the infimum is taken over all possible densities $p_{\mathbf{v}}$ of the input distributions allowed for the channel.

For a number of well-known noisy channel models, the channel blurredness may be evaluated analytically. A notable example is the classical additive white Gaussian noise (AWGN) channel, for which $\mathbf{u}_k = \mathbf{v}_k + \mathbf{n}_k$, where the noise $\{\mathbf{n}_k\}$ is a zero-mean white Gaussian process, and $\{\mathbf{n}_k\}$ and $\{\mathbf{v}_k\}$ are assumed to be independent. We impose a power constraint in the form

$$\lim_{k \rightarrow \infty} \frac{\mathbb{E} \sum_{i=0}^k \mathbf{v}_i^2}{k+1} \leq P.$$

For simplicity, we denote this constraint by $\mathbb{E} \mathbf{v}^2 \leq P$.

The following proposition presents the channel blurredness of an AWGN channel [9].

Proposition 1: The channel blurredness of an AWGN channel with noise variance N and power constraint $\mathbb{E} \mathbf{v}^2 \leq P$ is given by

$$B = \frac{1}{2} \log \left(1 + \frac{N}{P} \right). \quad (5)$$

It is well known that the channel capacity of the AWGN channel is given by [12]

$$C = \max_{p_{\mathbf{v}}: \mathbb{E} \mathbf{v}^2 \leq P} I(\mathbf{v}; \mathbf{u}) = \frac{1}{2} \log \left(1 + \frac{P}{N} \right).$$

Thus, for an AWGN channel, an explicit relationship can be established between channel blurredness and channel capacity, found as

$$B = \frac{1}{2} \log \left(1 + \frac{1}{2^{2C} - 1} \right). \quad (6)$$

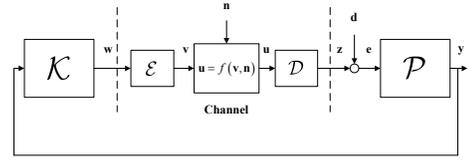


Fig. 1. A feedback system over an uplink noisy channel.

Clearly, when interpreted in the sense of channel capacity, the channel blurredness serves as a measure of poorness on the channel's quality.

For more sophisticated channels, however, no direct relationship may exist between channel blurredness and channel capacity. As such, the role of one cannot be superseded by the other. See [3] for a more detailed discussion.

We say that an asymptotically stationary process $\{\mathbf{x}_k\}$ is a power signal if $R_{\mathbf{x}}(k)$ is finite for any integer k and $S_{\mathbf{x}}(\omega)$ exists. For a power signal $\{\mathbf{x}_k\}$, the power norm can be defined as

$$\text{pow}(\mathbf{x}) = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\mathbf{x}}(\omega) d\omega} = \sqrt{R_{\mathbf{x}}(0)}.$$

Note that $\text{pow}(\mathbf{x})$ is a semi-norm.

We next introduce the power gain [9]. As the system gain induced by power signals, power gain represents the largest power amplification ratio from a system's input to output.

Definition 5: Consider a causal system \mathcal{F} . Let its input process $\{\mathbf{x}_k\}$ be a power signal. If the output process $\{\mathbf{y}_k\}$ is also a power signal, then the power gain of the system from $\{\mathbf{x}_k\}$ to $\{\mathbf{y}_k\}$ is defined as

$$\text{PG}(\mathcal{F}) \triangleq \sup_{\text{pow}(\mathbf{x}) \neq 0} \frac{\text{pow}(\mathbf{y})}{\text{pow}(\mathbf{x})}. \quad (7)$$

Power gain defines a viable theoretical notion with tangible practical relevance for a wide variety of systems. Indeed, it is a standard engineering practice to measure and use a system's input/output spectra for performance assessment. For a stable LTI system \mathcal{F} with transfer function $F(z)$, it is known that the power gain coincides with the \mathcal{H}_{∞} norm of the system,

$$\text{PG}(\mathcal{F}) = \|F(z)\|_{\infty} = \sup_{\omega} |F(e^{j\omega})|, \quad (8)$$

which is a well-established measure of performance.

III. PERFORMANCE LIMITATIONS

A. Networked Feedback Systems

Consider the system in Fig. 1. The plant \mathcal{P} is an LTI system with state-space model given by

$$\begin{bmatrix} \mathbf{x}_{k+1} \\ \mathbf{y}_k \end{bmatrix} = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_k \\ \mathbf{e}_k \end{bmatrix}, \quad (9)$$

where $\mathbf{x}_k \in \mathbb{R}^m$ is the state, $\mathbf{e}_k \in \mathbb{R}$ is the control input, and $\mathbf{y}_k \in \mathbb{R}$ the plant's output. The system matrices are $\mathcal{A} \in \mathbb{R}^{m \times m}$, $\mathcal{B} \in \mathbb{R}^{m \times 1}$, and $\mathcal{C} \in \mathbb{R}^{1 \times m}$. The initial state \mathbf{x}_0 is a random vector with a finite entropy $h(\mathbf{x}_0)$. The controller \mathcal{K} is assumed to be causal. We say that \mathcal{K} stabilizes

\mathcal{P} if $\sup_k \mathbb{E} [\mathbf{x}_k^T \mathbf{x}_k] < \infty$; that is, the closed-loop system is *mean-square stable*. The channel is assumed to be a general causal noisy channel with input $\{\mathbf{v}_k\}$, noise $\{\mathbf{n}_k\}$, and output $\{\mathbf{u}_k\}$. The encoder \mathcal{E} and the decoder \mathcal{D} are also assumed to be causal. Furthermore, we assume that $\{\mathbf{n}_k\}$, $\{\mathbf{d}_k\}$, \mathbf{v}_0 , \mathbf{x}_0 are mutually independent.

Our first main result is a Bode-type integral [9] that characterizes the trade-off of disturbance attenuation.

Theorem 1: Consider the system in Fig. 1. Suppose that $\{\mathbf{d}_k\}$ and $\{\mathbf{e}_k\}$ are asymptotically stationary, and that $h(\mathbf{d}_{0,\dots,k}) / (k+1)$ converges as $k \rightarrow \infty$. If the controller \mathcal{K} stabilizes the plant \mathcal{P} , then

$$\begin{aligned} \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \sqrt{\frac{S_{\mathbf{e}}(\omega)}{S_{\mathbf{d}}(\omega)}} d\omega &\geq J_{\infty}(\mathbf{e}) - J_{\infty}(\mathbf{d}) + I_{\infty}(\mathbf{n}; \mathbf{e}) \\ &+ \sum_{i=1}^m \max\{0, \log |\lambda_i(\mathcal{A})|\}, \end{aligned} \quad (10)$$

where $S_{\mathbf{d}}(\omega)$ and $S_{\mathbf{e}}(\omega)$ are the asymptotic power spectra of $\{\mathbf{d}_k\}$ and $\{\mathbf{e}_k\}$ respectively, and $\lambda_i(\mathcal{A})$, $i = 1, \dots, m$, denote the eigenvalues of matrix \mathcal{A} .

In the integral inequality (10), the term $J_{\infty}(\mathbf{e}) - J_{\infty}(\mathbf{d})$ quantifies the effect of non-Gaussian disturbance signals. When $\{\mathbf{d}_k\}$ is Gaussian, we have $J_{\infty}(\mathbf{e}) - J_{\infty}(\mathbf{d}) \geq 0$. Ignoring the noise effect further, that is, by discarding the term $I_{\infty}(\mathbf{n}; \mathbf{e}) \geq 0$, the inequality (10) is weakened to

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \log \sqrt{\frac{S_{\mathbf{e}}(\omega)}{S_{\mathbf{d}}(\omega)}} d\omega \geq \sum_{i=1}^m \max\{0, \log |\lambda_i(\mathcal{A})|\},$$

which gives the same result as that of [4] for systems without communication channels.

In the presence of a communication channel and under the assumption of a Gaussian disturbance $\{\mathbf{d}_k\}$, however, it was shown in [5], modulo to some additional assumptions, that

$$\begin{aligned} \frac{1}{2\pi} \int_{-\pi}^{\pi} \min \left\{ 0, \log \sqrt{\frac{S_{\mathbf{e}}(\omega)}{S_{\mathbf{d}}(\omega)}} \right\} d\omega \\ \geq \sum_{i=1}^m \max\{0, \log |\lambda_i(\mathcal{A})|\} - C_f, \end{aligned} \quad (11)$$

where C_f is the feedback capacity of the channel with one-step delay feedback. In particular, for an AWGN channel with one-step delay feedback, it can be shown that C_f coincides with the capacity C of the AWGN channel [12]. Under this circumstance, as noted in [14], the inequality (11) implies that disturbance attenuation can only be achieved under the condition

$$C_f > \sum_{i=1}^m \max\{0, \log |\lambda_i(\mathcal{A})|\},$$

that is, the capacity exceeds that required for feedback stabilization. The present integral inequality (10) goes further to show that when this is the case, the level of disturbance attenuation is constrained by a number of factors depending

on the Gaussianity of the disturbance signal, the channel noise effect, and the plant unstable poles.

Our following result presents a bound on power reduction [9], which characterizes the fundamental limit in disturbance attenuation of noisy networked feedback systems under power measure.

Theorem 2: Consider the system in Fig. 1. Let $\mathcal{T}_{\mathbf{de}}$ be the system from $\{\mathbf{d}_k\}$ to $\{\mathbf{e}_k\}$. Suppose that $\{\mathbf{d}_k\}$ and $\{\mathbf{e}_k\}$ are power signals, and that $h(\mathbf{d}_{0,\dots,k}) / (k+1)$ converges as $k \rightarrow \infty$. Furthermore, suppose that the decoder \mathcal{D} is injective. If \mathcal{K} stabilizes \mathcal{P} , then

$$\text{PG}(\mathcal{T}_{\mathbf{de}}) \geq 2^B \prod_{i=1}^m \max\{1, |\lambda_i(\mathcal{A})|\}. \quad (12)$$

In particular, if the channel is an AWGN channel with noise variance N , power constraint P , and channel capacity C , then

$$\begin{aligned} \text{PG}(\mathcal{T}_{\mathbf{de}}) &\geq \left(\sqrt{1 + \frac{N}{P}} \right) \prod_{i=1}^m \max\{1, |\lambda_i(\mathcal{A})|\}, \\ &= \left(\sqrt{\frac{2^{2C}}{2^{2C} - 1}} \right) \prod_{i=1}^m \max\{1, |\lambda_i(\mathcal{A})|\}. \end{aligned} \quad (13)$$

It is of interest to see that in (12) the channel noise has a particularly notable effect, which grows exponentially with the channel blurredness. When specialized to an AWGN channel, the lower bound can be interpreted in terms of the signal-to-noise ratio (SNR) of the channel. It can be seen from (13) that as the SNR P/N increases, the lower bound decreases. In the limit, when the SNR $\rightarrow \infty$, the lower bound approaches $\prod_{i=1}^m \max\{1, |\lambda_i(\mathcal{A})|\}$, i.e., the same lower bound corresponding to a noiseless channel.

In comparison, for a conventional LTI system without a communication channel, shown, e.g., in Fig. 2, the system $\mathcal{T}_{\mathbf{de}}$ corresponds to the sensitivity function

$$S(z) = \frac{1}{1 + P(z)K(z)},$$

where $P(z)$ and $K(z)$ are the transfer functions of the plant and the controller, respectively. In this case, under the condition that the closed-loop system is stable, the power gain coincides with the \mathcal{H}_{∞} norm. It is well known [15] that when the open-loop transfer function is strictly proper,

$$\|S(z)\|_{\infty} \geq \prod_{i=1}^m \max\{1, |\lambda_i|\}, \quad (14)$$

where λ_i , $i = 1, \dots, m$, denote the poles of $P(z)K(z)$.

B. Limits of the Feedback Mechanism

We now investigate performance bounds in variance minimization of general feedback systems. Consider the system in Fig. 2. Herein, all signals take scalar values as $\mathbf{d}_k, \mathbf{e}_k, \mathbf{z}_k, \mathbf{y}_k \in \mathbb{R}$. Assume that the plant \mathcal{P} is a strictly causal mapping, i.e., $\mathbf{y}_k = \mathcal{P}_k(\mathbf{e}_{0,\dots,k-1})$, for any $k \in \mathbb{N}$. Moreover, the controller \mathcal{K} is assumed to be causal, i.e., $\mathbf{z}_k = \mathcal{K}_k(\mathbf{y}_{0,\dots,k})$. Furthermore, $\{\mathbf{d}_k\}$ and \mathbf{z}_0 are assumed

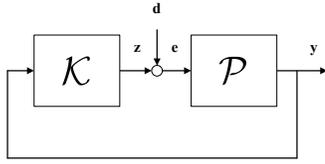


Fig. 2. A general feedback system.

to be independent. Note that if the plant is not strictly causal, then the controller should be assumed strictly causal so as to ensure the strict causality of the open-loop system, thus preventing $\{d_k\}$ and z_0 from being dependent.

The following theorem [10] exhibits that the asymptotic variance of the error is lower bounded by that of the disturbance, where the gain is found to be the Gaussianity-whiteness of the disturbance.

Theorem 3: Consider the general feedback system in Fig. 2. Suppose that $\{d_k\}$ and $\{e_k\}$ are asymptotically stationary. Then,

$$\lim_{k \rightarrow \infty} E[e_k^2] \geq \text{GW}_d \lim_{k \rightarrow \infty} E[d_k^2]. \quad (15)$$

Note that Theorem 3 is a very general result that holds for broad classes of feedback systems as long as the open-loop systems are strictly causal.

We next examine fundamental performance limitations of state estimation systems based on the bound given by (15). The results explicitly quantify how the estimation error is inherently bounded by the system and noise properties. Consider the estimation system depicted in Fig. 3. Herein, the system to be estimated can possibly be nonlinear and time varying with its state-space model given by

$$\begin{cases} \mathbf{x}_{k+1} = f_k(\mathbf{x}_k) + \mathbf{w}_k, \\ \mathbf{y}_k = h_k(\mathbf{x}_k) + \mathbf{v}_k, \end{cases}$$

where $\mathbf{x}_k \in \mathbb{R}^m$ is the state to be estimated, $\mathbf{y}_k \in \mathbb{R}$ is the system output, $\mathbf{w}_k \in \mathbb{R}^m$ is the process noise, and $\mathbf{v}_k \in \mathbb{R}$ is the measurement noise. In general, $\{\mathbf{w}_k\}$ and $\{\mathbf{v}_k\}$ are not necessarily white Gaussian. We employ the state estimator based on the state-space model of the system. Specifically, the estimator is given by

$$\begin{cases} \bar{\mathbf{x}}_{k+1} = f_k(\bar{\mathbf{x}}_k) + \mathbf{u}_k, \\ \bar{\mathbf{y}}_k = h_k(\bar{\mathbf{x}}_k), \\ \mathbf{e}_k = \mathbf{y}_k - \bar{\mathbf{y}}_k, \\ \mathbf{u}_k = K_k(\mathbf{e}_0, \dots, \mathbf{e}_k), \end{cases}$$

where $\bar{\mathbf{x}}_k \in \mathbb{R}^m$, $\bar{\mathbf{y}}_k \in \mathbb{R}$, $\mathbf{e}_k \in \mathbb{R}$, $\mathbf{u}_k \in \mathbb{R}^m$. Herein, the innovation $\{\mathbf{e}_k\}$ is processed by a general causal estimator $K_k(\cdot)$.

The next theorem [10] manifests the intrinsic limits of estimation systems in terms of variance minimization.

Theorem 4: Consider the system in Fig. 3. Suppose that $\{\mathbf{y}_k\}$ and $\{\mathbf{e}_k\}$ are asymptotically stationary. Then,

$$\lim_{k \rightarrow \infty} E[e_k^2] \geq \text{GW}_y \lim_{k \rightarrow \infty} E[\mathbf{y}_k^2]. \quad (16)$$

Theorem 4 shows that the variance of estimation error is lower bounded by the Gaussian-whiteness and variance of system output $\{\mathbf{y}_k\}$.

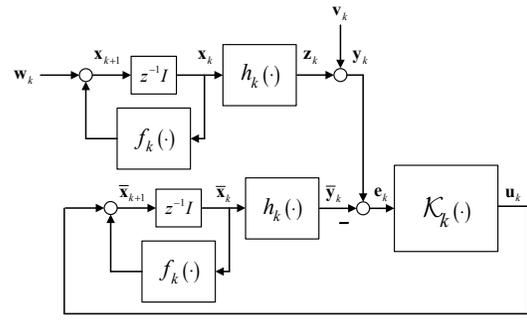


Fig. 3. A general state estimation system.

IV. CONCLUSION

This extended abstract presents a unifying information-theoretic framework to analyze the design limits and trade-offs imposed by communication channels on feedback control performance. This abstract also provides a cohesive treatment of performance limitation issues of generic feedback systems via an information-theoretic approach.

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