23rd International Symposium on Mathematical Theory of Networks and Systems
Hong Kong University of Science and Technology, Hong Kong, July 16-20, 2018

Optimal Steady-State Regulation of Multi-Agent Systems with Embedded Technique (Extended Abstract)

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I. Introduction

Multi-agent coordination has drawn much research interests due to the fast development of large-scale systems/networks, and multiple high-order agents have been widely discussed to deal with some practical coordination problems. As one fundamental problem of this topic, leader-following coordination has been widely studied [1]–[4]. In this problem, a (virtual) leader is often set up to generate the reference signals for each agent to follow, while this leader is usually given as a known dynamic system with possible unknown states. Then the main task is to determine the agents' controllers, which should only utilize local information, such that the resultant states or output trajectories of the agents can track the reference signal generated by the leader.

In this paper, we follow this line but consider a special case when the reference signal is not generated as the trajectory of an autonomous leader, but as the unknown optimal solution of distributed optimization problems. This type of problems arises naturally from many practical applications. For example, in a source seeking problem, we aim to control one or more agents with nonlinear dynamics to seek the extremum of some unknown signal field based on local signal measurements. Thus the reference (although a constant) is neither available in advance nor can be generated by an autonomous leader without real-time measurements and computations. Many other practical engineering applications have a similar feature that the reference signal is a (time-varying) maximum or minimum of some performance function, e.g., the design of anti-lock braking systems [5], optimal rendezvous of unmanned aerial vehicles [6], (optimal) frequency regulation in power grids [7].

II. Problem formulation

Consider a collection of agents of the form

$$\begin{aligned}\dot{x}_i &= A_i x_i + B_i u_i\\ y_i &= C_i x_i + D_i u_i, \quad i=1,\dots,N,\end{aligned}\tag{1}$$

where $x \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are state, input and output variables, respectively. Constant matrices A_i, B_i, C_i are with proper dimensions and the triple (C_i, A_i, B_i) is stabilizable and detectable.

Each agent has a local cost function $f_i\colon \mathbb{R} \to \mathbb{R}$, which is twice continuously differentiable with bounded Hessian, i.e., there exist finite positive constants $\underline{h}_i, \bar{h}_i$ satisfying $\underline{h}_i \leq \nabla^2 f_i(s) \leq \bar{h}_i$ for all $s \in \mathbb{R}$. For a distributed design, the communication topology among agents is described by a weighted undirected graph $\mathscr{G}$, and agent i has its own local cost function $f_i(\cdot)$, which is only known to itself and cannot be shared globally in the multi-agent network. Denote $y = \mathrm{col}(y_1, \dots, y_N)$, and we associate these agents with the following contained optimization problem:

$$\begin{aligned}\text{minimize} \quad & f(y) = \textstyle\sum_{i=1}^N f_i(y_i)\\ \text{subject to} \quad & \textstyle\sum_{i=1}^N y_i = \sum_{i=1}^N d_i\end{aligned}\tag{2}$$

where d_i is a constant. As usual, we assume there exists a finite optimal solution $y^* = \mathrm{col}(y_1^*, \dots, y_N^*)$ to (2).

The optimal steady-state regulation problem is formulated as follows. *Given a graph $\mathscr{G}$, agent (1), and the optimization problem (2), find a distributed control for u_i such that each agent reaches a steady-state with an associated output $y_i^* \triangleq C_i x_i^*$ by only its own local data and exchanged information with its neighbors, i.e., $\lim_{t\to+\infty} y_i = y_i^*$ for $i = 1, \dots, N$.*

Several technical assumptions are made to achieve a cooperation for solving this problem.

Assumption 1: The information sharing graph $\mathscr{G}$ is undirected and connected.

Assumption 2: For each $i = 1, \dots, N$, there exist constant matrices X_{i1}, X_{i2}, and U_i satisfying

$$\begin{aligned}0 &= A_i X_i + B_i U_i\\ 1 &= C_i X_i + D_i U_i.\end{aligned}$$

The formulated problem is essentially an asymptotic regulation problem where the reference is determined by the optimization problem (2). Hence the above condition known as regulator equations is crucial in our design.

III. Main results

The most challenging part to solve this problem is the high-order structure of agents including its coupling with the distributed optimization requirements. To avoid this difficulty, we employ the embedded scheme proposed in [8] and solve the distributed optimal steady-state regulation problem.

Briefly, we first introduce an optimal signal generator by considering the same optimization problem for "virtual" single integrators, in order to asymptotically reproduce the optimal solution y^* by a signal z_i. Then, by taking z_i as an output reference signal for the high-order agents and embedding this generator in the feedback loop via a steady-state regulator for system (1). In this way, the optimal steady-state regulation problem is divided into two simpler subproblems: a) conventional distributed optimization design and b) asymptotic regulation for general linear agents.

This work was supported by the National Nature Science Foundation of China under Grant 61503033.

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A. Optimal signal generator

A optimal signal generator is essentially a dynamic system reproducing the optimal solution of (2) (for single integrators). For this part, we propose the following system:

$$\begin{aligned} \dot{z}_i &= -\nabla f_i(z_i) + \lambda_i \\ \dot{\lambda}_i &= -\lambda_i^v - w_i^v + d_i - z_i \\ \dot{w}_i &= \lambda_i^v \end{aligned} \tag{3}$$

where $\lambda_i^v \triangleq \sum_{j=1}^N a_{ij}(\lambda_i - \lambda_j)$, $w_i^v \triangleq \sum_{j=1}^N a_{ij}(w_i - w_j)$.

Its effectiveness is presented as follows.

Lemma 1: Suppose Assumption 1 hold. The algorithm (3) will exponentially solve the optimization problem (2), i.e., z_i converges to y_i^* exponentially fast as $t \to +\infty$ for $i = 1, \ldots, m$.

B. Steady-state regulator

With the above optimal signal generator, we seek a proper steady-state regulator and then embed it into this regulator to solve the distributed optimal steady-state regulation problem.

By the stabilizability of (A_i, B_i), there exists a matrix K_{i1} such that $A_i + B_i K_{i1}$ is Hurwitz. Denote $K_{i2} \triangleq U_i - K_{i1} X_i$. The first main theorem of this paper as follows.

Theorem 1: Suppose Assumptions 1-2 hold. The following algorithm solves the optimal steady-state regulation problem for this linear multi-agent system:

$$\begin{aligned} u_i &= K_{i1} x_i + K_{i2} z_i \\ \dot{z}_i &= -\nabla f_i(z_i) + \lambda_i \\ \dot{\lambda}_i &= -\lambda_i^v - w_i^v + d_i - z_i \\ \dot{w}_i &= \lambda_i^v. \end{aligned} \tag{4}$$

Furthermore, y_i exponentially converges to y_i^* as $t \to \infty$.

Next, let us consider the case when only the output variables of each agent can be obtained because it may be difficult to get or measure all the state variables in some situations. Since the optimal signal generator is independently implemented, we only have to focus on the tracking part.

To solve the problem, we consider an output feedback version of the proposed embedded control by attaching a Luenberger observer. Take gain matrices L_i such that $A_i + L_i C_i$ is Hurwitz ($i = 1, \ldots, N$), then we have the following theorem.

Theorem 2: Suppose Assumptions 1-2 hold. The following algorithms solve the optimal steady-state regulation problem for this linear multi-agent system:

$$\begin{aligned} u_i &= K_{i1} \xi_i + K_{i2} z_i \\ \dot{\xi}_i &= A_i \xi_i + B_i u_i + L_i (C_i \xi_i - y_i) \\ \dot{z}_i &= -\nabla f_i(z_i) + \lambda_i \\ \dot{\lambda}_i &= -\lambda_i^v - w_i^v + d_i - z_i \\ \dot{w}_i &= \lambda_i^v. \end{aligned} \tag{5}$$

Furthermore, y_i exponentially converges to y_i^* as $t \to \infty$.

C. Extensions with real-time gradients

In some cases, we can not have the cost function f_i itself and only its real-time gradient is available. Then, the above control law will not be implementable. Note that there will always be some error between $\nabla f_i(y_i)$ and $\nabla f_i(z_i)$ when $z_i \neq y_i$, and this error will be smaller if the tracking subsystem evolves in a faster time scale. Thus, we use some high-gain technique to solve this problem.

We only present the state feedback results.

Theorem 3: Suppose Assumptions 1-2 hold and the system (1) is minimum-phase with relative degree r_i for $i = 1, \ldots, N$. There exists a constant $\varepsilon^* > 0$, such that the following algorithm solves the distributed optimal steady-state regulation problem for this linear multi-agent system with real-time gradients when $\varepsilon \in (0, \varepsilon^*)$:

$$\begin{aligned} u_i &= -\frac{1}{\varepsilon^{r_i}} [c_{i0}(y_i - z_i) + \sum_{k=1}^{r_i - 1} \varepsilon^k c_{ik} C_i A_i^k x_i] - \frac{C_i A_i^{r_i}}{C_i A_i^{r_i - 1} B_i} x_i \\ \dot{z}_i &= -\nabla f_i(y_i) + \lambda_i \\ \dot{\lambda}_i &= -\lambda_i^v - w_i^v + d_i - z_i \\ \dot{w}_i &= \lambda_i^v \end{aligned} \tag{6}$$

where the parameters c_{ik} are chosen such that the polynomial $\sum_{k=0}^{r_i - 1} c_{ik} s^k + s^{r_i}$ is Hurwitz ($i = 1, \ldots, N$).

The distributed optimal steady-state regulation problem allows us to take both distributed optimization and steady-state regulation of high-order plants into consideration. In fact, the formulated problem extended the well-studied resource allocation problem to high-order multi-agent systems [9], [10]. When $N = 1$, the formulated problem can be taken as an asymptotic regulation problem, where the steady state optimizes a given objective function. This problem has been called optimalizing control or extremum seeking in some literature [5], [11], [12]. Thus, we actually consider its distributed extensions with gradient information for high-order unstable plants.

Based on the above analysis, it is worthwhile to mention that the embedded control scheme provides a general and promising formulation to solve these problems in a unified way. In fact, the design complexities brought by high-order dynamics are decoupled from the optimization task. By solving the two simpler subproblems, the optimal output consensus problem can be solved via constructive controllers. Also, this embedded control framework enjoys a large flexibility in choosing optimal signal generators and tracking controllers, and therefore, admits various optimization algorithms.

IV. Conclusions

This paper has investigated the optimal steady-state regulation problem for general linear multi-agent systems. An embedded control scheme has been proposed and applied to solve this problem based on the introduction of an optimal signal generator. The proposed algorithms have been proved to converge to the optimal solution exponentially with different information. In fact, many challenging optimal output consensus problems remain to be done, including the cases with nonlinear agents and various uncertainties.

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