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Persistent-hold Consensus Control of First-order Multi-agent Systems with Intermittent Communication

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Abstract— **Under intermittent communication, a persistent-hold consensus algorithm is proposed to solve the consensus problem of first-order multi-agent systems. Based on matrix theory and graph theory, consensus conditions are obtained for the agents converging to the asymptotic consensus under fixed topology and switching topologies, respectively. Numerical simulations show the correctness of theoretical results.**

I. INTRODUCTION

In the past decades, tremendous research efforts have been devoted for distributed coordination control of multi-agent systems for its widely engineering applications in formation control of UAVs, sensor networks, smart grid, etc.

As a fundamental issue of coordination control, consensus problem requires that the outputs of several agents reach an agreement on the state of interest via locally exchanging information. Consensus problem has been widely analyzed and synthesized for multi-agent systems under different communication constraints including packet loss, noises, time delays, switching topologies, etc [1], [2], [3], [4], [5].

Moreover, intermittent communication considered in this paper means that the agents communicate with its neighbors at intermittent time intervals, but not continuously. To deal with intermittent communication, current works have mainly focused on the corresponding intermittent control which requires the control input to be driven by normal consensus coordination parts with communication and driven by zero without communication [6]. For first-order multi-agent systems with nonlinear dynamics, Wen et al. analyzed the consensus problem with intermittent communication and obtained sufficient consensus conditions with a fixed strongly connected topology [7]. Besides, consensus problem of second-order multi-agent systems with intermittent communication has received more extensive investigations. With intermittent dynamical consensus algorithms, consensus convergence conditions of nonlinear and linear second-order multi-agent systems without and with time delays have been obtained by constructing proper Lyapunov functions [8], [9], [10], [11], [12]. Wen et al. [13] designed intermittent consensus algorithms for the multi-agent systems with agents modelled by general linear dynamics, and consensus conditions on communication rate were obtained based on matrix theory and switching systems theory. For second-order multi-agent systems, Liu et al. [14] modified the usual intermittent control by a pulse-modulated intermittent control that combined the impulsive control and sampled control, some necessary and sufficient conditions have been obtained based on discretization approaches and stability theory [14]. Actually, the intermittent control has also been widely used in the stabilization control of nonlinear systems [15] and the synchronization control of complex networks [16], [17].

Motivated by the aforementioned intermittent control and the usual sampled-hold control [19], [20], [21], [22], we design a persistent-hold consensus algorithm to solve the consensus problem of first-order multi-agent systems under intermittent communication. By using matrix theory and graph theory, consensus conditions are obtained for the agents converging to the asymptotic consensus under fixed topology that has a spanning tree, firstly. Then, consensus conditions are also gained for the agents under switching topologies, in which the union of topologies consecutively across bounded and non-overlapping time intervals has a spanning tree.

II. PROBLEM FORMULATION

A. Agents' Dynamics and Topology

Consider the first-order dynamic agents given by

$$\dot{x}_i(t) = u_i(t), i = 1, \cdots, n, \tag{1}$$

where $x_i(t) \in R$ and $u_i(t) \in R$ are the state and the control input of agent i respectively.

A weighted digraph $G = (V, E, A)$ of order n consists of a set of vertices $V = \{1, \cdots, n\}$, a set of edges $E \subseteq V \times V$ and a weighted adjacency matrix $A = [a_{ij}] \in R^{n\times n}$ with $a_{ij} \geq 0$. A directed edge from i to j in G is denoted by $e_{ij} = (i, j) \in E$, which means that the node j can obtain information from the node i. Assume $a_{ji} > 0 \Leftrightarrow e_{ij} \in E$ and $a_{ii} = 0$ for all $i \in V$. The set of neighbors of node i is denoted by $N_i = \{j \in V : (j, i) \in E\}$. The Laplacian matrix of the digraph G is defined as $L = D - A = [l_{ij}] \in R^{n\times n}$, where $D = \mathrm{diag}\{\sum_{j=1}^{n} a_{ij}, i = 1, \cdots, n\}$ is the degree matrix. In the digraph G, a directed path from node i_1 to node i_s is a sequence of ordered edges of the form $(i_1, i_2), \cdots, (i_{s-1}, i_s)$ where $i_j \in V$. A digraph is said to have a spanning tree, if there exists a node such that there is a directed path from this node to every other node.

A matrix $C = [c_{ij}] \in R^{n\times r}$ is nonnegative if all its elements c_{ij} are nonnegative. If a nonnegative matrix $C \in R^{n\times r}$ satisfies $C1_r = 1_n$, then it is said to be (row) stochastic. A stochastic

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matrix $B \in R^{n\times n}$ is said to be indecomposable and aperiodic (*SIA*) if $\lim_{m\to\infty} B^m = 1_n f^T$ where $f \in R^n$. In this paper, $1_n = [1,1,\cdots,1]^T$, and I_n denotes a $n\times n$ identity matrix.

Lemma 1. [18] Let $P_1, P_2, \cdots, P_k \in R^{n\times n}$ be a finite set of *SIA* matrices with the property that for each sequence $P_{i_1}, P_{i_2}, \cdots, P_{i_j}$ with positive length, the matrix product $P_{i_j} P_{i_{j-1}} \cdots P_{i_1}$ is *SIA*. Then, for each infinite sequence $P_{i_1}, P_{i_2}, \cdots$, there exists a vector $f \in R^n$ such that $\lim_{j\to\infty} P_{i_j} P_{i_{j-1}} \cdots P_{i_1} = 1_n f^T$.

B. Intermittent Communication and Persistent-hold Control

In reality, the communication links between neighboring agents are intermittently connected for various constraints including obstruction, communication range, and so on. Aperiodically intermittent communication is illustrated in Fig. 1, and each time interval $[t_k, t_{k+1}]$ contains communicating interval h_{k1} and the left interval of communication loss.

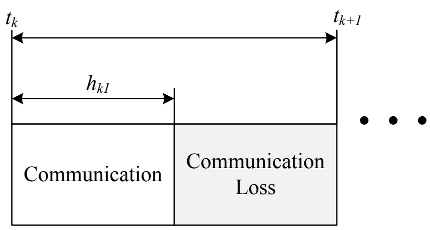


Fig. 1. Intermittent Communication.

In this paper, we assume that the interconnection topology of agents is fixed across the time interval $[t_k, t_{k+1}]$. For the multi-agent systems under intermittent communication, usual intermittent control, in which the control input is driven by normal consensus coordination parts with communication and driven by zero without communication, has been widely proposed to deal with the consensus seeking problem.

Different from intermittent control strategy, we propose a persistent-hold control strategy shown in Fig. 2. The time interval $[t_k, t_{k+1}]$ is divided into three parts composed of $[t_k, t_k + h_{k1}]$, $[t_k + h_{k1}, t_k + h_{k2}]$ and $[t_k + h_{k2}, t_{k+1}]$. In the time interval $[t_k, t_k + h_{k1}]$, the control input to be driven by persistent control, which is same as the usual intermittent control. In the communication loss interval $[t_k + h_{k1}, t_{k+1}]$, the control input is also driven by hold control for an assigned time interval $[t_k + h_{k1}, t_k + h_{k1} + h_{k2}]$ when the communication begins to be unconnected, and the control input is assumed to be zero for the left time interval $[t_k + h_{k1} + h_{k2}, t_{k+1}]$ after the hold control.

Then, the consensus algorithm based on the persistent-hold control strategy is designed as

$$u_i(t) = \begin{cases} \sum_{j\in N_i(k)} a_{ij}(k)(x_j(t) - x_i(t)), \\ \quad t \in (t_k, t_k + h_{k1}); \\ \sum_{j\in N_i(k)} a_{ij}(k)(x_j(t_k + h_{k1}) - x_i(t_k + h_{k1})), \\ \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ 0, \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}), \end{cases} \tag{2}$$

where $h_{k1} > 0, h_{k2} > 0, k = 0,1,2,\cdots$, $N_i(k)$ is the set of agent i's neighbors, $a_{ij}(k) > 0$, $j \in N_i(k)$ is the coupling weights.

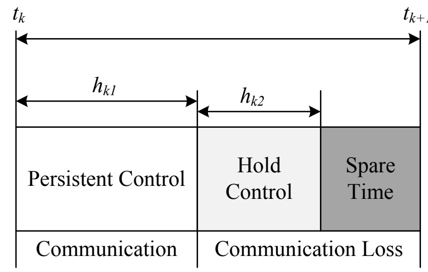


Fig. 2. Persistent-hold control strategy.

Remark 1. Compared with usual intermittent control strategy, the persistent-hold control strategy in (2) has an extra part defined as hold control, and we will demonstrate that the extra hold control can improve the control performance.

III. CONSENSUS SEEKING UNDER FIXED TOPOLOGY

In this section, we focus on the fixed topology that the coupling weights and the interconnections across all the communicating intervals are identical. Thus, the algorithm (2) with fixed topology turns to be

$$u_i(t) = \begin{cases} \sum_{j\in N_i} a_{ij}(x_j(t) - x_i(t)), \\ \quad t \in (t_k, t_k + h_{k1}); \\ \sum_{j\in N_i} a_{ij}(x_j(t_k + h_{k1}) - x_i(t_k + h_{k1})), \\ \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ 0, \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}). \end{cases} \tag{3}$$

With algorithm (3), the closed-loop form of agents (1) is given by

$$\dot{x}_i(t) = \begin{cases} \sum_{j\in N_i} a_{ij}(x_j(t) - x_i(t)), \\ \quad t \in (t_k, t_k + h_{k1}); \\ \sum_{j\in N_i} a_{ij}(x_j(t_k + h_{k1}) - x_i(t_k + h_{k1})), \\ \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ 0, \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}). \end{cases} \tag{4}$$

Formulate the above system (4) in a compact-vector form as

$$\dot{x}(t) = \begin{cases} -Lx(t), \quad t \in (t_k, t_k + h_{k1}); \\ -Lx(t_k + h_{k1}), \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ 0, \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}), \end{cases} \tag{5}$$

where $x(t) = [x_1(t), \cdots, x_n(t)]^T$. By computation, then, we get

$$x(t) = \begin{cases} e^{-L(t-t_k)} x(t_k), \\ \quad t \in (t_k, t_k + h_{k1}); \\ x(t_k + h_{k1}) - (t - (t_k + h_{k1}))Lx(t_k + h_{k1}), \\ \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ x(t_k + h_{k1} + h_{k2}), \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}). \end{cases}$$

Hence, we obtain

$$x(t_{k+1}) = (I - h_{k2}L)e^{-Lh_{k1}} x(t_k). \tag{6}$$

In this section, we will consider the following topology.

Assumption 1. The interconnection topology of the multi-agent systems (1) has a spanning tree.

With Assumption 1, we get the following lemma in [2].

Lemma 2. [2] 0 is a simple eigenvalue of the Laplacian matrix L, and $L1_n = 0$ with $1_n = [1, \cdots, 1]^T$, if and only if the digraph $G = (V, E, A)$ has a spanning tree.

With Assumption 1, the eigenvalues of the Laplacian matrix L are denoted by $\lambda_1 = 0, \lambda_i, i = 2 \cdots, n$ with $\mathrm{Re}(\lambda_i) > 0, i = 2 \cdots, n$.

Now, we get the following consensus conditions.

Theorem 1. Consider the multi-agent systems (4) with a fixed topology satisfying Assumption 1. The agents (4) converge to the consensus asymptotically, if

$$\sup_{k=1,2,\cdots,\infty} |(1 - h_{k2}\lambda_i)\mathrm{e}^{-h_{k1}\lambda_i}| < 1, i = 2, \cdots, n, \tag{7}$$

where $\lambda_i, i = 2, \cdots, n$ are the nonzero eigenvalues of L.

Proof. With Assumption 1, there exists a matrix $Q \in R^{n\times n}$ transforming the Laplacian matrix L into a Jordan form as

$$Q^{-1}LQ = J = \begin{bmatrix} 0 & 0 \\ 0 & \hat{J} \end{bmatrix}, \tag{8}$$

where $Q = [q_1, q_2, \cdots, q_n]$ with $q_i \in R^n$, $q_1 = [1, 1, \cdots, 1]^T$ is the eigenvector of $\lambda_1 = 0$, and $\hat{J} \in R^{(n-1)\times(n-1)}$ is in the Jordan form.

Let $z(t_k) = Q^{-1}x(t_k)$, and we get

$$z(t_{k+1}) = \Psi(k)z(t_k) \tag{9}$$

with

$$\Psi(k) = (I - h_{k2}J)\mathrm{e}^{-h_{k1}J}. \tag{10}$$

It follows from (8) that

$$\begin{bmatrix} z_1(t_{k+1}) \\ \hat{z}(t_{k+1}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & (I - h_{k2}\hat{J})\mathrm{e}^{-h_{k1}\hat{J}} \end{bmatrix} \begin{bmatrix} z_1(t_k) \\ \hat{z}(t_k) \end{bmatrix}, \tag{11}$$

where $z(t_k) = \begin{bmatrix} z_1(t_k) \\ \hat{z}(t_k) \end{bmatrix}$.

Conditions (7) guarantees that $\lim_{k\to\infty} \hat{z}(t_k) = 0$, and $\lim_{k\to\infty} z(t_k) = \mathrm{diag}\{1, 0, , \cdots, 0\}z(t_0)$. Hence,

$$\lim_{k\to\infty} x(t_k) = Q\mathrm{diag}\{1, 0, , \cdots, 0\}z(t_0) = [1, 1, \cdots, 1]^T c,$$

where $c = z_1(t_0)$, i.e., the agents (4) converge to the consensus asymptotically. Theorem 1 is proved. □

Remark 2. From the proof of Theorem 1, consensus convergence rate of the agents (4) is determined by $\sup_{k=1,2,\cdots,\infty} |(1 - h_{k2}\lambda_i)\mathrm{e}^{-h_{k1}\lambda_i}|, i = 2, \cdots, n$, and it is evident that the consensus convergence can be speeded up by choosing proper time intervals of hold control.

Example 1. We consider a multi-agent network composed of nine first-order agents given by (1), and the fixed interconnection topology shown in Fig. 3 has a spanning tree obviously. For simplicity, all the adjacent weights are chosen as 0.2, e.g., $a_{14} = a_{14} = a_{21} = a_{32} = a_{45} = a_{53} = a_{58} = a_{63} = a_{74} = a_{78} = a_{89} = a_{96} = 0.2$, and the eigenvalues of L are $\lambda_1 = 0, \lambda_2 = 0.4, \lambda_3 = 0.1119 + \mathrm{j}0.1373, \lambda_4 = 0.1119 - \mathrm{j}0.1373, \lambda_5 = 0.2631 + \mathrm{j}0.2101, \lambda_6 = 0.2631 - \mathrm{j}0.2101, \lambda_7 = 0.45, \lambda_8 = 0.4, \lambda_9 = 0.4$.

To compare our persistent-hold control algorithm with the usual intermittent control algorithm, we consider the periodically intermittent communications, i.e., the time intervals as $t_{k+1} - t_k = 0.6$(s) and the communication intervals as $h_{k1} = 0.2$(s). According to the conditions (7) in Theorem 1, we get the bound of holding control interval as $h_{k2} \leq 0.4$(s). Under usual intermittent control, the consensus converging time is 121.93(s) when $\max_{i=2,\cdots,n} |x_i - x_1| < 10^{-2}$. Then, we the consensus converging times of our proposed algorithm with different hold intervals (see Table I). Evidently, our persistent-hold consensus control have much faster converging rate than usual intermittent consensus control.

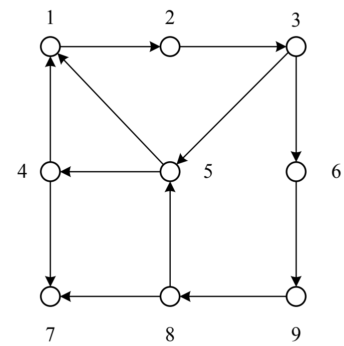


Fig. 3. Topology of nine agents.

TABLE I
CONVERGING TIME UNDER PERSISTENT-HOLD CONTROL

Holding interval h_{k2}(s)	Converging time (s)
0.05	97.82
0.10	81.67
0.15	70.25
0.20	62.52
0.25	54.89
0.30	49.55
0.35	45.20
0.40	41.54

Besides, we consider the more general case, and choose the time intervals as $t_{k+1} - t_k \in (0.6, 0.7)$(s), $h_{k1} \in (0.2, 0.3)$(s) and $h_{k2} \in (0.15, 0.35)$(s). Thus, the conditions (7) in Theorems 1 hold, and the agents converge to an asymptotic consensus (see Fig. 4).

IV. SWITCHING TOPOLOGIES

In this section, we study the consensus problem of agents (1) with switching topologies. In $t \in [t_k, t_{k+1})$, we assume that the topology of agents (1) is described by $G(k)$, which is fixed, and the corresponding Laplacian matrix is $L(k)$.

With switching topologies, the agents (1) with algorithm (2) are given by

$$\dot{x}_i(t) = \begin{cases} \sum_{j\in N_i(k)} a_{ij}(k)(x_j(t) - x_i(t)), \\ \quad t \in (t_k, t_k + h_{k1}); \\ \sum_{j\in N_i(k)} a_{ij}(k)(x_j(t_k + h_{k1}) - x_i(t_k + h_{k1})), \\ \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ 0, \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}). \end{cases} \tag{12}$$

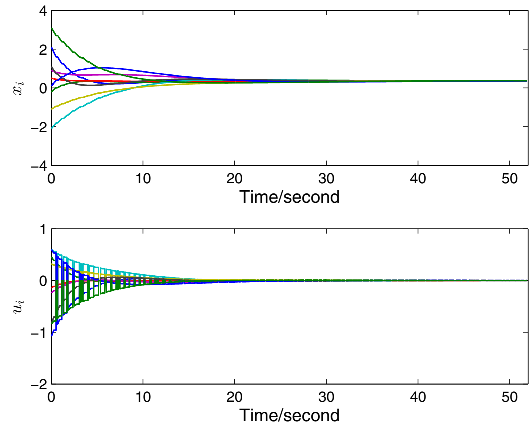


Fig. 4. Consensus convergence under fixed topology.

Rewrite (12) in a compact-vector form as

$$\dot{x}(t) = \begin{cases} -L(k)x(t), \quad t \in (t_k, t_k + h_{k1}); \\ -L(k)x(t_k + h_{k1}), \\ \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ 0, \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}), \end{cases} \tag{13}$$

and we obtain

$$x(t) = \begin{cases} \mathrm{e}^{-L(k)(t-t_k)}x(t_k), \\ \quad t \in (t_k, t_k + h_{k1}); \\ x(t_k + h_{k1}) - (t - (t_k + h_{k1}))L(k)x(t_k + h_{k1}), \\ \quad t \in (t_k + h_{k1}, t_k + h_{k1} + h_{k2}); \\ x(t_k + h_{k1} + h_{k2}), \quad t \in (t_k + h_{k1} + h_{k2}, t_{k+1}). \end{cases}$$

Hence, we obtain

$$x(t_{k+1}) = \Phi(k)x(t_k), \tag{14}$$

where

$$\Phi(k) = (I - h_{k2}L(k))\mathrm{e}^{-L(k)h_{k1}}.$$

Before presenting the results, we assume that the following prerequisite about the coupling weights and the hold intervals of the agents (12) is satisfied:

Prerequisite 1. $h_{k2}\sum_{j\in N_i(k)} a_{ij}(k) < 1, i = 1, \cdots, n$.

Lemma 3. With Prerequisite 1, $\Phi(k)$ is a row-stochastic matrix with positive diagonal entries, and the topology of Laplacian matrix $I - \Phi(k)$ contains all the edges of $G(k)$.

Proof. Let $S(k) = I - h_{k2}L(k)$, and $S(k)$ is obviously a row-stochastic matrix with positive diagonal entries with Prerequisite 1. The topology of Laplacian matrix $I - S(k)$ has the same edges as $G(k)$.

From the Lemma 2.6 in [23], $\mathrm{e}^{-h_{k1}L(k)}$ is also a row-stochastic matrix with positive diagonal entries, so $(I - h_{k2}L(k))\mathrm{e}^{-L(k)h_{k1}}$ must be a row-stochastic matrix with positive diagonal entries. Besides,

$$\begin{aligned} \Phi(k) &= S(k)\mathrm{e}^{-\frac{h_{k1}}{h_{k2}}(I-S(k))} \\ &= \mathrm{e}^{-\frac{h_{k1}}{h_{k2}}}S(k)\mathrm{e}^{\frac{h_{k1}}{h_{k2}}S(k)} \end{aligned}$$

Obviously, $\exists\delta > 0$, $\mathrm{e}^{-\frac{h_{k1}}{h_{k2}}}S(k)\mathrm{e}^{\frac{h_{k1}}{h_{k2}}S(k)} - \delta S(k)$ is a non-negative matrix, i.e., the topology of Laplacian matrix $I - \Phi(k)$ contains all the edges of $G(k)$. Lemma 3 is proved. □

Lemma 4. Under Prerequisite 1, if the union of a set of topologies $G(k_1), G(k_1+1), \cdots, G(k_2)$ of agents (12) has a spanning tree with the positive integers k_1 and k_2 satisfying $k_2 > k_1$, then, $\prod_{k_1}^{k_2}\Phi(k)$ is SIA.

Proof. Under Prerequisite 1, the topologies associated with $I - \Phi(k_1), I - \Phi(k_1+1), \cdots, I - \Phi(k_2)$ are defined as $\bar{G}(k_1), \bar{G}(k_1+1), \cdots, \bar{G}(k_2)$. Since the union of the digraphs $G(k_1), G(k_1+1), \cdots, G(k_2)$ of agents (12) has a spanning tree, it follows from Lemma 3 and the assumption in Lemma 4 that the union of the digraphs $\bar{G}(k_1), \bar{G}(k_1+1), \cdots, \bar{G}(k_2)$ has a spanning tree. Then, Lemma 4 can be proved similar to the proof of Lemma 7 in [24]. Lemma 4 is proved. □

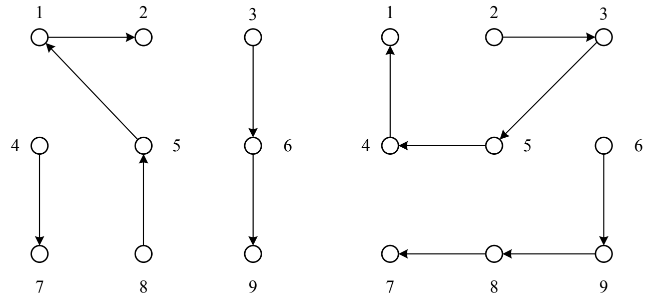
Theorem 2. Prerequisite 1 holds for the multi-agent systems (12) with switching topologies. The agents in (12) converge to an asymptotic consensus, if there exists an infinite sequence of uniformly bounded, non-overlapping time intervals $[t_{k_i}, t_{k_{i+1}}), i = 1, 2, \cdots, \infty, 0 < k_{i+1} - k_i \le d, d \in Z_+$, starting at $k_1 = 0$, and the union of the topologies of n agents across each interval $[t_{k_i}, t_{k_{i+1}})$ has a spanning tree.

Proof. From (14), we get $x(t_{k+1}) = \Phi(k)\cdots\Phi(k_{i+1})\prod_{m=1}^{i}\Psi(m)x(t_0)$, where $\Psi(i) = \Phi(k_{i+1}-1)\Phi(k_{i+1}-2)\cdots\Phi(k_i)$. Since the union of n agents' interconnection topologies across $[t_{k_i}, t_{k_{i+1}})$ has a spanning tree, $\Psi(i)$ is *SIA* from Lemmas 3 and 4. Because $0 < k_{m+1} - k_m \le d$ and the possible topologies of the n agents (12) are finite, the set of all possible $\Psi(i)$ is finite. According to Lemma 1, we get $\prod_{m=1}^{\infty}\Psi(m) = 1_n f^T$, where $f \in R^n$ is a constant vector. In addition, $\Phi(k)$ is a stochastic matrix with Prerequisite 1, so we obtain $\lim_{k\to\infty} x(k+1) = 1_n f^T x(t_0)$. Hence, $\lim_{k\to\infty} x_i(t_k) = f^T x(t_0), i = 1, \cdots, n$, i.e., the agents (12) achieve an asymptotic stationary consensus. Theorem 2 is proved. □

Example 2. We take into account a multi-agent network composed of nine first-order agents given by (1), and The interconnection topology of the system is switched between topology 1 and topology 2 every 0.6(s) in Fig. 5, and the union of topology 1 and topology 2 has a spanning tree. The adjacent weights are also chosen as 0.2 for the two topologies, and the time intervals are set as $t_{k+1} - t_k = 0.6$(s), $h_{k1} \in (0.2, 0.3)$(s) and $h_{k2} \in (0.15, 0.35)$(s). Evidently, Prerequisite 1 and the conditions in Theorem 2 hold. Hence, the agents (12) reach the consensus asymptotically (see Fig. 6).

V. CONCLUSION

In this paper, we investigate the first-order multi-agent systems under intermittent communications, and design a persistent-hold consensus algorithm to deal with the consensus convergence problem. Our proposed algorithm consists of the persistent control in communication interval and the hold control in the part of communication loss interval. Under the fixed topology, consensus conditions are gained for the agents converging to the consensus asymptotically by using



(a) Topology 1 (b) Topology 2

Fig. 5. Switching topologies

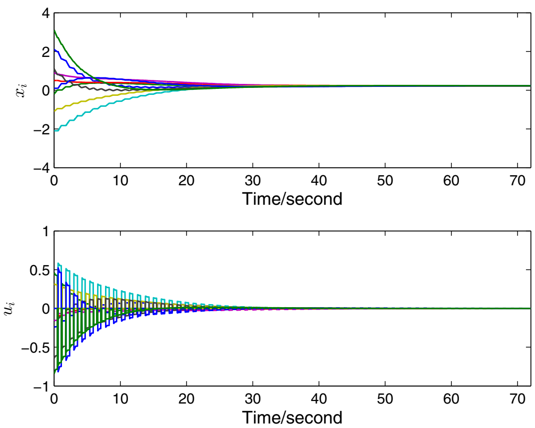


Fig. 6. Consensus convergence under switching topologies.

matrix theory and graph theory, and the consensus convergence rate can be speeded up by choosing proper hold control intervals. In addition, sufficient consensus conditions are also obtained for the agents reaching the asymptotic consensus under switching topologies, of which the union consecutively across bounded and non-overlapping time intervals has a spanning tree. In our future research work, furthermore, we will applied the persistent-hold control strategy into the consensus problem of multi-agent systems with more complicated agents' dynamics.

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